

Solving a System Algebraically: For each problem, do the following.

- 1) Solve each equation for y.
- 2) Set the two equations equal to each other and solve for x.
- 3) Plug your x values into either one of the original equations to solve for the y values.
- 4) State your solutions as coordinate points.

1) $y = -2(x-2)^2 + 8$
 $y = (x-2)^2 + 5$

$(3, 6) (1, 6)$

$$\begin{aligned} -2(x-2)^2 + 8 &= (x-2)^2 + 5 \\ -2(x^2 - 4x + 4) + 8 &= (x^2 - 4x + 4) + 5 \\ -2x^2 + 8x - 8 + 8 &= x^2 - 4x + 9 \\ -2x^2 + 8x &= x^2 - 4x + 9 \\ +2x^2 - 8x & \quad +2x^2 - 8x \\ \hline 0 &= 3x^2 - 12x + 9 \\ 0 &= 3(x^2 - 4x + 3) \\ 0 &= 3(x-3)(x-1) \\ x &= 3, x = 1 \end{aligned}$$

$x = 3: y = (3-2)^2 + 5 = (1)^2 + 5 = 1 + 5 = 6$
 $x = 1: y = (1-2)^2 + 5 = (-1)^2 + 5 = 1 + 5 = 6$

3) $x + y = 6$ $y = -x + 6$
 $y = -(x-4)^2 + 4$

$$\begin{aligned} -x + 6 &= -(x-4)^2 + 4 \\ -x + 6 &= -(x^2 - 8x + 16) + 4 \\ -x + 6 &= -x^2 + 8x - 16 + 4 \\ -x + 6 &= -x^2 + 8x - 12 \\ +x^2 - 8x + 12 &= +x^2 - 8x + 12 \\ \hline x^2 - 9x + 18 &= 0 \\ (x-6)(x-3) &= 0 \\ x &= 6, 3 \end{aligned}$$

$x = 6: 6 + y = 6 \quad y = 0$
 $x = 3: 3 + y = 6 \quad y = 3$

$(6, 0) (3, 3)$

2) $y = \frac{1}{2}x^2 + 4$
 $y = 3x^2 - 6$

$$\frac{1}{2}x^2 + 4 = 3x^2 - 6$$

$3 = \frac{6}{2}$
 $-\frac{1}{2}$

$$\begin{aligned} \frac{1}{2}x^2 + 4 &= 3x^2 - 6 \\ -\frac{1}{2}x^2 & \quad -\frac{1}{2}x^2 \\ \hline 4 &= 5x^2 - 6 \\ +6 & \quad +6 \\ \hline \frac{2}{5}(10) &= \left(\frac{5}{2}x^2\right) \frac{2}{5} \\ 4 &= x^2 \\ x &= \pm 2 \end{aligned}$$

$x = 2: y = 3(2)^2 - 6 = 3(4) - 6 = 12 - 6 = 6$
 $x = -2: y = 3(-2)^2 - 6 = 3(4) - 6 = 6$

$(2, 6) (-2, 6)$

4) $y = x^2$
 $y = x + 2$

$$\begin{aligned} x^2 &= x + 2 \\ x^2 - x - 2 &= 0 \\ (x-2)(x+1) &= 0 \\ x &= 2, -1 \end{aligned}$$

$x = 2: y = 2 + 2 = 4$
 $x = -1: y = -1 + 2 = 1$

$(2, 4) (-1, 1)$

$$5) \begin{cases} x+y=1 \\ y=-x+1 \\ y=-(x+1)^2+4 \end{cases}$$

$$-x+1=-(x+1)^2+4$$

$$-x+1=-(x^2+2x+1)+4$$

$$-x+1=-x^2-2x-1+4$$

$$\frac{-x+1}{+x^2+2x-3} = \frac{-x^2-2x+3}{+x^2+2x-3}$$

$$x^2+x-2=0$$

$$(x+2)(x-1)=0$$

$$x=-2, 1$$

$$x=-2 \quad -2+y=1 \quad y=3$$

$$x=1 \quad 1+y=1 \quad y=0$$

$$\boxed{(-2, 3) \quad (1, 0)}$$

$$6) \begin{cases} y=-x^2-3 \\ y=x^2-5 \end{cases}$$

$$-x^2-3=x^2-5$$

$$\frac{-x^2-3}{+x^2} = \frac{x^2-5}{+x^2}$$

$$\frac{-3}{+5} = \frac{-5}{2+1}$$

$$\frac{2}{2} = \frac{2x}{2}$$

$$1 = x^2$$

$$x=1 \quad y=(1)^2-5=1-5=-4$$

$$x=-1 \quad y=(-1)^2-5=1-5=-4$$

$$\boxed{(1, -4) \quad (-1, -4)}$$

$$7) \begin{cases} y=x^2-4 \\ y=3x \end{cases}$$

$$x^2-4=3x$$

$$x^2-3x-4=0$$

$$(x-4)(x+1)=0$$

$$x=4, -1$$

$$x=4 \quad y=3(4)=12$$

$$x=-1 \quad y=3(-1)=-3$$

$$\boxed{(4, 12) \quad (-1, -3)}$$

$$8) \begin{cases} y=(x-1)^2+3 \\ 2x+y=5 \end{cases} \quad y=-2x+5$$

$$(x-1)^2+3=-2x+5$$

$$x^2-2x+1+3=-2x+5$$

$$x^2-2x+4=-2x+5$$

$$\frac{x^2-4}{+2x-5} = \frac{-2x+5}{+2x-5}$$

$$x^2-1=0$$

$$x^2=1$$

$$x=\pm 1$$

$$x=1 \quad \begin{cases} 2(1)+y=5 \\ 2+y=5 \end{cases} \quad y=3$$

$$x=-1 \quad \begin{cases} 2(-1)+y=5 \\ -2+y=5 \end{cases} \quad y=7$$

$$\boxed{(1, 3) \quad (-1, 7)}$$