Algebra 2 & Trigonometry Unit 3 Test Review

Polynomial Functions and Equations

Name: Master E

Date: _____ Block: _____

1-8: Operations with Polynomials – Simplify each expression completely. *Leave no negative exponents in final solutions.*

1. $x^7 \bullet x^3 \bullet x$

x^{7+3+1}

x^{11}

2. $(5x^4)^{-2}$

$5^{-2}x^{-4\cdot 2}$

$5^{-2}x^{-8}$

$\frac{1}{25x^8}$

3. $\frac{5p^5q^{-3}}{25p^2q^{-5}}$

$\frac{5p^5q^5}{25p^2q^3}$

$\frac{p^3q^2}{5}$

4. $(-4x^0)^2(2x^3y^3)^3$

$(-4^2 \cdot x^{0\cdot 2})(2^3x^9y^9)$

$16 \cdot 1 \cdot 8x^9y^9$

$128x^9y^9$

5. $(4x^3+5x-7x^2)+(-2x^3+5x^2-7y^2)$

$-2x^3 \quad +5x^2$

$2x^3 \quad -2x^2$

$2x^3-2x^2+5x-7y^2$

6. $(-3x^2+7x+23)-(-8x^2-5x+13)$

$+8x^2+5x-13$

$5x^2+12x+10$

7. $(4x-3)^2$

$(4x-3)(4x-3)$

$-12x \quad -12x$

$16x^2-24x+9$

8. $(3x+5)(9x^2-15x+25)$

$3x(9x^2-15x+25)+5(9x^2-15x+25)$

$27x^3-45x^2+75x+45x^2-75x+125$

$27x^3+125$ sum of cubes!

9-10: Dividing Polynomials - Use long division in #9 and synthetic division in #10.

9. $(15v^3+8v^2-21v+6)\div(5v-4)$

$3v^2+4v-1$

$5v-4 \;|\; 15v^3+8v^2-21v+6$

$-15v^3+12v^2$

$20v^2-21v$

$-20v^2+16v$

$-5v+6$

$+5v+4$

2

$3v^2+4v-1+\frac{2}{5v-4}$

10. $(-2x^3+15x^2-10x+3)\div(x+3)$

$-3 \;|\; -2 \;\; 15 \;\; -10 \;\; 3$

$\downarrow \;\; 6 \;\; -63 \;\; 219$

$-2 \;\; 21 \;\; -73 \;\; 222$

$-2x^2+21x-73+\frac{222}{x+3}$

11-13: Polynomial Functions – Find each indicated value for $p(x) = -4x^2 + 1$ and $m(x) = x^3 + 2x^2 - 3$.

11. p(-5)

$-4(-5)^2+1$

$-4(25)+1$

$-100+1$

-99

12. m($3a^3$)

$(3a^3)^3+2(3a^3)^2-3$

$3^3a^9+2(3^2a^6)-3$

$27a^9+2(9a^6)-3$

$27a^9+18a^6-3$

13. p(a + 2)

$-4(a+2)^2+1$

$-4(a^2+4a+4)+1$

$-4a^2-16a-16+1$

$-4a^2-16a-15$

14-15: Analyzing Graphs of Polynomial Functions

14. $f(x) = x^2(x-6)(x^2-5x-6)(x+5)(x^2-36)$

$x^2(x-6)(x-6)(x+1)(x+5)(x+6)(x-6)$

$x^2(x-6)^3(x+1)(x+5)(x+6)$

a. What is the degree of this function? 8

b. Is the degree even or odd? even

c. What is the maximum number of turns? 7

d. How many zeros will this function have? 8

e. List the zeros of this function. 0, 0, 6, 6, 6, -1, -5, -6

f. Are there any points of tangency to the x-axis? Yes: 0

g. Are there any terrace points at the x-axis? Yes: 6

h. As $x \to -\infty$, $f(x) \to$ +∞ and as $x \to +\infty$, $f(x) \to$ +∞

i. Estimate the x coordinates for the local maximum(s).

btw. -5 & -1 and at 0

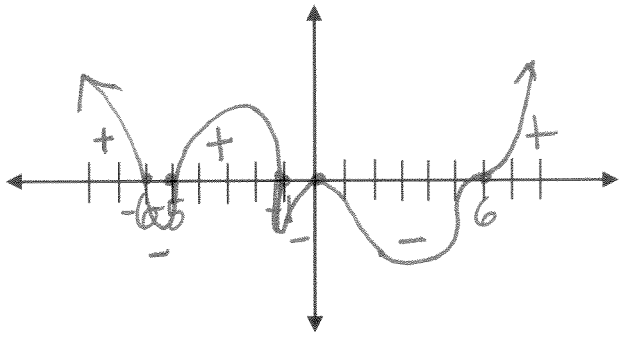
j. Estimate the x coordinates for the local minimum(s).

btw. -6 & -5, btw. -1 & 0, and btw 0 & 6

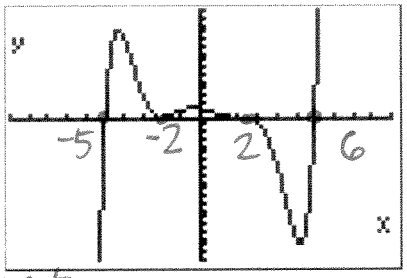
k. Complete the sign change chart for the function.

	x^2	$(x-6)^3$	$x+1$	$x+5$	$x+6$	$f(x)$
-7	+	−	−	−	−	+
-5.5	+	−	−	−	+	−
-2	+	−	−	+	+	+
-.5	+	−	+	+	+	−
3	+	−	+	+	+	−
7	+	+	+	+	+	+

l. Sketch the graph of the function below.
Be sure to label the units on the x-axis!



15. Use the graph to complete a – f below.



a. Is the function even or odd? How do you know?

odd b/c the end behavior is in opposite directions

b. Is the leading coefficient positive or negative? How do you know?

positive b/c as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to +\infty$, $f(x) \to +\infty$

c. What is the minimum degree of the function? How do you know?

7 b/c there are 7 real zeros when considering there is 1 terrace & 1 tangent.

d. Estimate the real zeros of the function.

-5, -2, -2, 2, 2, 2, 6

e. Are there any zeros of even multiplicity? How do you know?

Yes. -2 is a tangent to the x-axis.

f. Are there any zeros of odd multiplicity? How do you know?

Yes. There is a terrace at 2 because it "swerves".

16-21: Solving Polynomial Equations – Solve each polynomial equation over the set of complex numbers by FACTORING.
Provide exact solutions only. Therefore, all irrational solutions must be in simplified radical form (no decimal answers).

16. $f(x) = 9x^4 - 100$

$(3x^2+10)(3x^2-10)=0$

$3x^2+10=0 \quad 3x^2-10=0$

$3x^2=-10 \quad 3x^2=10$

$x^2=\frac{-10}{3}\cdot\frac{\sqrt{3}}{\sqrt{3}} \quad x^2=\frac{10}{3}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

You have to rationalize the denominator!

factored form: $f(x)=(3x^2+10)(3x^2-10)$

zeros: $\pm i\frac{\sqrt{30}}{3}, \pm\frac{\sqrt{30}}{3}$

17. $18x^3 + 84x^2 + 98x = 0$

$2x(9x^2+42x+49)=0$

$2x(3x+7)^2=0$

$2x=0 \quad 3x+7=0$

$x=0 \quad 3x=-7 \Rightarrow x=\frac{-7}{3}$

factored form: $2x(3x+7)^2=0$

zeros: $0, -\frac{7}{3}, -\frac{7}{3}$

18. $x^3 - 25x = 7x^2 - 175$

$x^3-7x^2-25x+175=0$

$x^2(x-7)-25(x-7)$

$(x-7)(x^2-25)=0$

$x-7=0$

$x=7$

factored form: $(x-7)(x+5)(x-5)=0$

zeros: $7, -5, 5$

19. $x^5 + 30x = 11x^3$

$x^5-11x^3+30x=0$

$x(x^4-11x^2+30)=0$

$30 \mid -11$

$-5,-6$

$x(x^2-5)(x^2-6)=0$

$x=0 \quad x^2-5=0 \quad x^2-6=0$

$x^2=5 \quad x^2=6$

factored form: $x(x^2-5)(x^2-6)=0$

zeros: $0, \pm\sqrt{5}, \pm\sqrt{6}$

20. $y = 3x^3 + 81$

$3(x^3+27)=0$

$3(x+3)(x^2-3x+9)=0$

$x+3=0 \quad x=\frac{3\pm\sqrt{9-4(9)}}{2}$

$x=-3 \quad \frac{3\pm\sqrt{-27}}{2}=\sqrt{-27}$

$i\sqrt{9\cdot3}$

$3i\sqrt{3}$

factored form: $3(x+3)(x^2-3x+9)=0$

zeros: $-3, \frac{3\pm3i\sqrt{3}}{2}$

21. $48x^4 - 27x^2 = 0$

$3x^2(16x^2-9)=0$

$3x^2(4x+3)(4x-3)=0$

$3x^2=0 \quad 4x+3=0 \quad 4x-3=0$

$x=0 \quad 4x=-3 \quad 4x=3$

$x=-\frac{3}{4} \quad x=\frac{3}{4}$

factored form: $3x^2(4x+3)(4x-3)=0$

zeros: $0, 0, \pm\frac{3}{4}$

22-23: The Remainder and Factor Theorems – Given a polynomial and one if its factors, find the remaining factors of the polynomial.

22. $(x^3 - x^2 + x + 14);(x+2)$

$-2 \mid 1 \quad -1 \quad 1 \quad 14$

$\downarrow \quad -2 \quad 6 \quad -14$

$1 \quad -3 \quad 7 \quad 0$

$(x+2)(x^2-3x+7)$

23. $(5x^3 - 17x^2 + 6x);(x-3)$

$3 \mid 5 \quad -17 \quad 6 \quad 0$

$\downarrow \quad 15 \quad -6$

$5 \quad -2 \quad 0$

$(x-3)(5x^2-2x)$

$(x-3)(x)(5x-2)$

$x(x-3)(5x-2)$

24-27: Roots and Zeros – Write a polynomial function of least degree that has the given features. Write the function in its factored form ONLY!

24. fifth degree; zeros: x = 4, x = -5, terrace at -1; y-intercept: (0, -20)

$y=a(x-4)(x+5)(x+1)^3$

$-20=a(0-4)(0+5)(0+1)^3$

$-20=a(-4)(5)(1)$

$-20=-20a$

$1=a$

$f(x)=(x-4)(x+5)(x+1)^3$

25. zeros: $x=3, x=\frac{1}{7}, x=-\frac{2}{3}$; y-intercept: 12

$y=a(x-3)(7x-1)(3x+2)$

$12=a(-3)(-1)(2)$

$12=6a$

$2=a$

$f(x)=2(x-3)(7x-1)(3x+2)$

26. zeros: $x=\pm 5i$, $x=-7$; y-intercept: (0, 175)

$y=a(x^2+25)(x+7)$

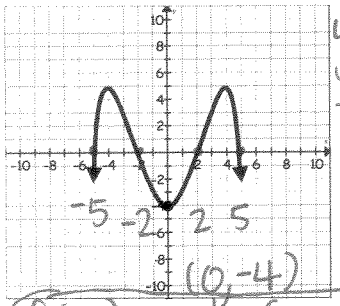
$175=a(25)(7)$

$175=175a$

$1=a$

$f(x)=(x^2+25)(x+7)$

27.



$y=a(x+5)(x+2)(x-2)(x$

$-4=a(5)(2)(-2)(-5)$

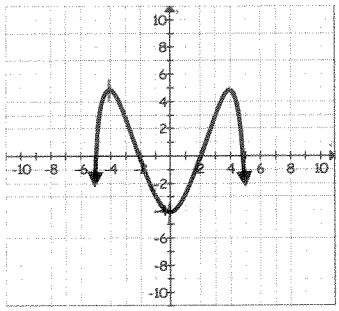
$-4=100a$

$-\frac{1}{25}=a$

$f(x)=-\frac{1}{25}(x+5)(x+2)(x-2)(x-5)$

28: Behavior of Functions – Determine the intervals on which the function is increasing or decreasing as indicated.

28.



a. What is the behavior of the function appear to be on the interval (-4, 0)?

It is decreasing

b. On which interval does this graph appear to be increasing?

$(-\infty,-4)$ and $(0,4)$

c. On which interval does this graph appear to be decreasing?

$(-4,0)$ and $(4,\infty)$

29-30: Find all of the roots of the function by depressing the polynomial.

Provide exact solutions only. Therefore, all irrational solutions must be in simplified radical form (no decimal answers).

29. $f(x)=x^3-7x^2+16x-10$ calc. $x=1$ 1 real, 2 imag.

1	1	-7	16	-10
	↓	1	-6	10
	1	-6	10	0

$(x-1)(x^2-6x+10)$

$\frac{6\pm\sqrt{36-4(10)}}{2}$

$\frac{6\pm\sqrt{-4}}{2}=\frac{6\pm 2i}{2}$

$=3\pm i$

$x=1, 3\pm i$

30. $f(x)=x^4+x^3+2x^2+4x-8$ calc. $x=-2$, $x=1$

-2	1	1	2	4	-8
	↓	-2	2	-8	8
1	1	-1	4	-4	0
	↓	1	0	4	
	1	0	4	0	

$(x+2)(x-1)(x^2+4)=0$

$x^2+4=0$

$x^2=-4$

$x=\pm 2i$

$x=-2, 1, \pm 2i$