5-6 The Quadratic Formula & The Discriminant

Master Eq #1

The graph of a function can be used to find the x-intercepts (zeros), but many times, the solutions are not clear. We can find the exact solutions of a quadratic function by solving for x.

SOLVING QUADRATIC EQUATIONS BY FACTORING:

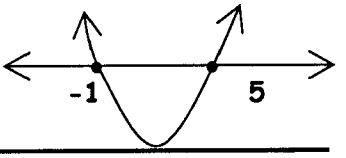
1. Put the equation in general form.
2. Factor the polynomial and set y = 0.
3. Set each factor equal to zero and solve.
4. Check your answers on the calculator.

$y = x^2 - 4x - 5$

$(x - 5)(x + 1) = 0$

$x - 5 = 0$ → $\boxed{x = 5}$

$x + 1 = 0$ → $\boxed{x = -1}$



SOLVING QUADRATIC EQUATIONS BY TAKING THE SQUARE ROOT:

1. Put the equation in general form and set y = 0.
2. Complete the square.
3. Move the constant to the other side.
4. Take the square root of both sides.
5. Write 2 equations and solve for x.

$y = x^2 - 4x - 5$

$x^2 - 4x - 5 = 0$

$x^2 - 4x + \_\_\_ - 5 - \_\_\_ = 0$

$x^2 - 4x + \underline{4} - 5 - \underline{4} = 0$

$(x - 2)^2 - 9 = 0$

$(x - 2)^2 = 9$

$\sqrt{(x - 2)^2} = \sqrt{9}$

$x - 2 = \pm 3$ → $x - 2 = 3$ → $\boxed{x = 5}$; $x - 2 = -3$ → $\boxed{x = -1}$

SOLVING QUADRATIC EQUATIONS USING THE QUADRATIC FORMULA:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Put the equation in general form and set y = 0
2. Determine the values for *a*, *b* and *c*.
3. Substitute these values into the quadratic formula
4. Simplify all values in parentheses and solve for x.

$x^2 - 4x - 5 = 0$

$a = \underline{1}\ b = \underline{-4}\ c = \underline{-5}$

$$x = \frac{-(-4) \pm \sqrt{-4^2 - 4\ 1\ -5}}{2\ 1}$$

$$x = \frac{4 \pm \sqrt{16 - (-20)}}{2}$$

$$x = \frac{4 \pm \sqrt{16 + 20}}{2} = \frac{4 \pm \sqrt{36}}{2}$$

$$x = \frac{4 \pm 6}{2}$$

$\frac{4 + 6}{2}$ → $\frac{10}{2}$ → $\boxed{x = 5}$

$\frac{4 - 6}{2}$ → $\frac{-2}{2}$ → $\boxed{x = -1}$

ConceptSummary Solving Quadratic Equations

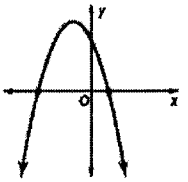
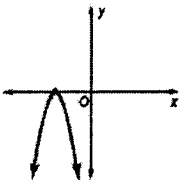
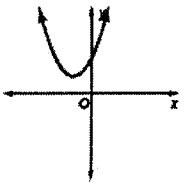
Method	Can be Used	When to Use
graphing	sometimes	Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.
factoring	sometimes	Use if the constant term is 0 or if the factors are easily determined. Example $x^2 - 7x = 0$
Square Root Property	sometimes	Use for equations in which a perfect square is equal to a constant. Example $(x - 5)^2 = 18$
completing the square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is even. Example $x^2 + 6x - 14 = 0$
Quadratic Formula	always	Useful when other methods fail or are too tedious. Example $2.3x^2 - 1.8x + 9.7 = 0$

Discriminant = $b^2 - 4ac$

Can be used to determine the number and type of roots of a quadratic equation.

KeyConcept Discriminant

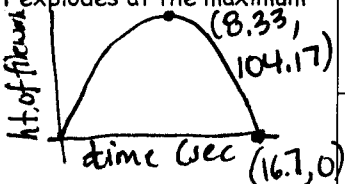
Consider $ax^2 + bx + c = 0$, where a, b, and c are rational numbers and $a \neq 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	
$b^2 - 4ac < 0$	2 complex roots	

Word Problem Review: **A firework's distance d in meters from the ground is given by $d = -1.5t^2 + 25t$, where t is the number of seconds after the firework has been lit.**

A. How many seconds have passed since the firework was lit when the firework explodes if it explodes at the maximum height of its path?

8.33 seconds



B. What is the height of the firework when it explodes?

104.17 meters

C. How much time will it take for the firework remains to hit the ground?

16.7 seconds

Examples: Find the following:

A. Discriminant b^2-4ac

B. Describe the roots

C. Solve the equation

1. $x^2 + 12x = -32$

$x^2+12x+32=0$

$\frac{-12 \pm \sqrt{16}}{2}$

$\frac{-12 \pm 4}{2} = \frac{-12+4}{2}, \frac{-12-4}{2}$

$\frac{-8}{2}, \frac{-16}{2} = \{-8, -4\}$

A. $12^2-4(1)(32)$ $=16$

B. 2 real rational

C.

2. $12x - 5 = 2x^2 + 1$

$2x^2-12x+6=0$

$\frac{12 \pm \sqrt{96}}{4}$

$\frac{12 \pm 4\sqrt{6}}{4}$

C. $\{3 \pm \sqrt{6}\}$ exact ans. or $\{.55, 5.45\}$ approx. ans.

A. $(-12)^2-4(2)(6)$ $=96$

B. 2 real irrational

3. $-2x^2 = -2x + 3$

$0=2x^2-2x+3$

$\frac{2 \pm \sqrt{-20}}{4}$

$\frac{2 \pm 2i\sqrt{5}}{4}$

C. $\frac{1 \pm i\sqrt{5}}{2}$ or $\frac{1}{2} \pm \frac{i\sqrt{5}}{2}$

A. $(-2)^2-4(2)(3)$ $=-20$

B. 2 complex (imaginary)

4. $3x^2 + 8x = 35$

$3x^2+8x-35=0$

$\frac{-8 \pm \sqrt{484}}{6}$

$\frac{-8 \pm 22}{6}$

$\frac{-8+22}{6}, \frac{-8-22}{6} = \frac{14}{6}, \frac{-30}{6}$

C. $\{-5, \frac{7}{3}\}$

A. $8^2-4(3)(-35)$ $=484$

B. 2 real rational