

TEST REVIEW: Unit 3 - Polynomial Functions and Equations

**Learning Target 1** I can simplify polynomial expressions and apply the properties of exponents.

1-4: Simplify each expression.

1. Simplify  $(-4x^5)^2(7x^0)$ .  $16x^{10} \cdot 7$   
 $112x^{10}$

2. Simplify  $\frac{2a^3b^{-6}}{6a^2b^{-8}}$ . Assume no variable equals zero.  
 $\frac{2a^3b^8}{6a^2b^6} = \frac{ab^2}{3}$

3. Simplify  $(-4x^3y^5)(3x^2y^6 + 5x^4y)$   
 $-12x^5y^{11} - 20x^7y^6$

4. Simplify  $\frac{30x^8 - 3x^4 + 27x^3}{3x^2}$   
 $10x^6 - x^2 + 9x$

**Learning Target 2** I can divide polynomials using polynomial long division and synthetic division and apply the properties of the Remainder and Factor Theorems.

5-6: Circle the best solution.

5. Given that  $(x - 2)$  is a factor of  $x^4 + 5x^2 - 36$ , which of the following is the remainder when  $x^4 + 5x^2 - 36$  is divided by  $(x - 2)$ ?

- A. -2
- B. -18
- C. 2
- D. 0

$$\begin{array}{r} 2 \overline{) 1050-36} \\ \underline{20} \phantom{00} \\ 250 \phantom{00} \\ \underline{40} \phantom{00} \\ 180 \phantom{00} \\ \underline{36} \phantom{00} \\ 144 \phantom{00} \\ \underline{288} \phantom{00} \\ 144 \phantom{00} \\ \underline{288} \phantom{00} \\ 0 \phantom{00} \end{array}$$

6. Find the quotient.  $(x^3 + 2x^2 - 5x - 6) \div (x + 3)$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -5 & -6 \\ & \downarrow & -3 & 3 & 6 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$x^2 - x - 2$

**Learning Target 3** I can describe the characteristics and behavior of a polynomial function given its graph.

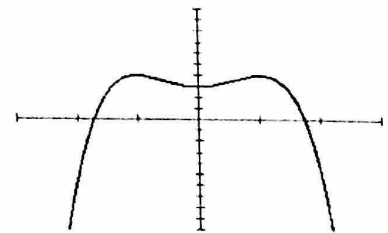
7-10: Circle the best solution.

7. Which is the maximum number of turns for a function of the form:  $f(x) = ax^5 + bx^2 + cx + d$

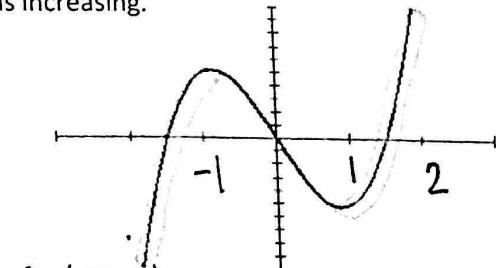
- A. 4
  - B. 3
  - C. 2
  - D. 1
- Always 1 less!

8. Which is the degree and sign of the leading coefficient of the polynomial function shown at right?

- A. degree 4, positive
- ~~B. degree 3, positive~~
- C. degree 4, negative
- ~~D. degree 3, negative~~

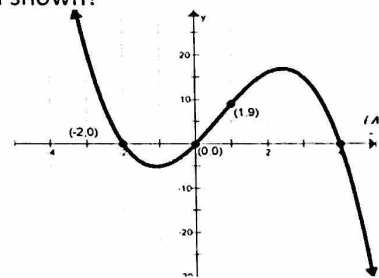


9. Approximate the intervals for which the function is increasing.



- A.  $(-\infty, \infty)$
- B.  $(-1, 1)$
- C.  $(-\infty, -1), (1, \infty)$
- D.  $(-\infty, 0), (0, \infty)$

10. Which best describes the end behavior of the function shown?



- A. As  $x \rightarrow -\infty, f(x) \rightarrow -\infty$  and as  $x \rightarrow \infty, f(x) \rightarrow -\infty$
- B. As  $x \rightarrow -\infty, f(x) \rightarrow \infty$  and as  $x \rightarrow \infty, f(x) \rightarrow \infty$
- C. As  $x \rightarrow -\infty, f(x) \rightarrow -\infty$  and as  $x \rightarrow \infty, f(x) \rightarrow \infty$
- D. As  $x \rightarrow -\infty, f(x) \rightarrow \infty$  and as  $x \rightarrow \infty, f(x) \rightarrow -\infty$

11. Use the graph at the right to complete a - h.

a. Is the function of even or odd degree? How do you know?

EVEN because the arrows are going in the same direction.

b. Is the leading coefficient positive or negative? How do you know?

Positive because it is going  $\uparrow$  in the end.

c. Estimate the real zeros of the function.

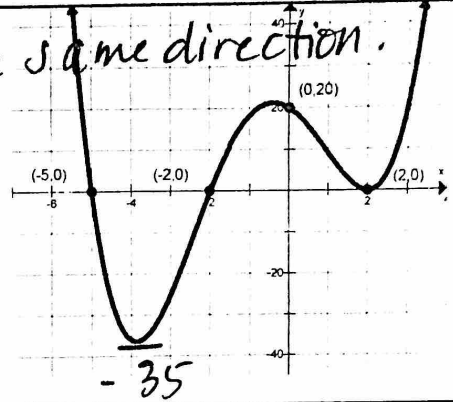
-5, -2, 2, 2

d. Does there appear to be any zeros of multiplicity (tangent, terrace)? If so, where?

Tangent at 2

f. Is the function increasing or decreasing on the interval  $x: (2, \infty)$ .

Increasing



g. What is the smallest possible degree of the function? How do you know?

4 because it has 4 zeroes

h. State the domain and range of the function.

Domain:  $(-\infty, \infty)$  Range (estimate):  $[-35, \infty)$

12. Complete a - f for  $f(x) = x(x+3)^2(x-1)(x^2-25)$ . =  $x(x+3)^2(x-1)(x+5)(x-5)$

a. What is the degree of this function?

HINT: the function is in factored form.

6

b. What is the maximum number of turns in this function?

5

c. How many total zeros does this function have?

6

d. What are the zeros of this function?

0, -3, -3, 1, -5, 5

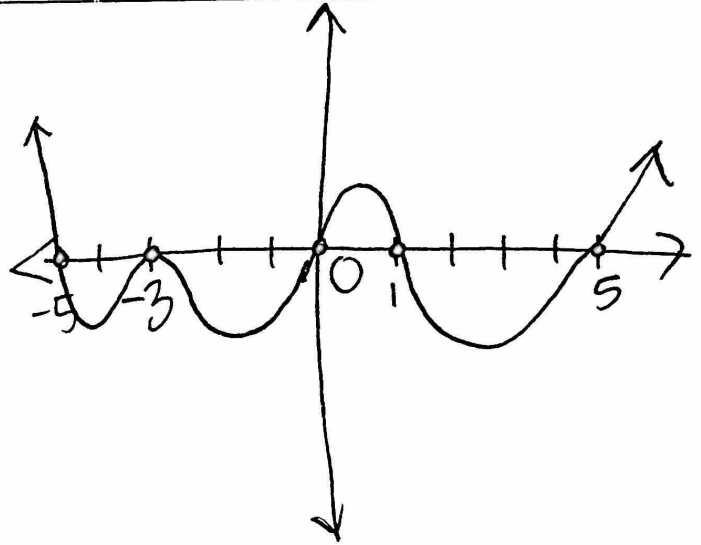
6 total

e. Are there any points of tangency to the x-axis? If so, at what x-value?

-3

f. Are there any terrace points at the x-axis? If so, where?

No



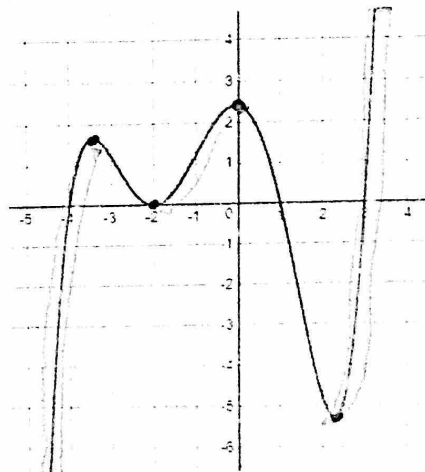
13. Use the graph on the right to answer a-d.

a. As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -\infty$  and as  $x \rightarrow \infty$ ,  $f(x) \rightarrow +\infty$

b. There are two minimums at approximately  $x = -2$  and  $x = 2.2$

c. There are two maximums at approximately  $x = -3.5$  and  $x = 0$

d. Circle each interval that is increasing:  $(-3.5, -2)$   $(2.3, \infty)$   $(-2, 0)$   $(0, 2.3)$   $(-\infty, -3.5)$



**Learning Target 4:** I can write the equation of a polynomial function given its zeros/roots or graph.

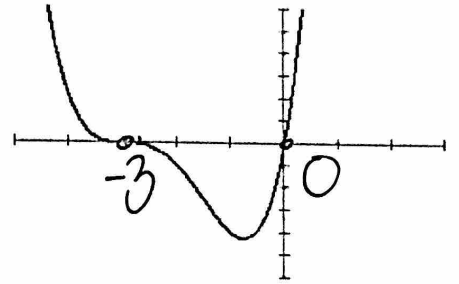
14-15: Circle the best solution.

14. Which defines a polynomial function with zeros 4, -1, and -2?

- A.  $f(x) = x(x-4)(x+1)(x+2)$   
 B.  $f(x) = x(x+4)(x-1)(x-2)$   
 C.  $f(x) = (x-4)(x+1)(x+2)$   
 D.  $f(x) = (x+4)(x-1)(x-2)$

15. Which of the following could be the function for the graph at the right?

- ~~A.~~  $p(x) = -x(x-3)^2$   
 B.  $p(x) = x(x+3)$   
~~C.~~  $p(x) = -x(x-3)^3$   
 D.  $p(x) = x(x+3)^3$

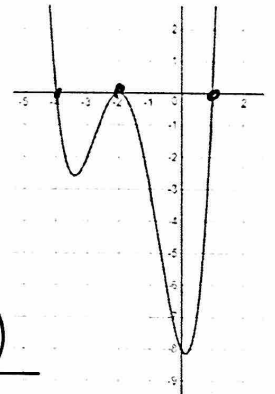


16: Write the equation of the fifth-degree polynomial function in its **factored form** that has a tangent point at 3, and other roots of  $\pm 2i$  and 7. (assume  $a=1$ )

16.  $y = (x-3)^2(x^2+4)(x-7)$

17. Write the polynomial function in **factored form** for the graph.

Assume  $a = \frac{1}{2}$



17.  $f(x) = \frac{1}{2}(x+4)(x+2)(x-1)$

**Learning Target 5:** I can solve a higher degree polynomial equation over the set of complex numbers by factoring.

18. Factor completely:

$3x^2 + 11x - 4$

$$\begin{array}{r} -12 \quad | \quad 11 \\ 12 \quad -1 \quad | \end{array}$$
  

$$3x^2 + 12x - x - 4$$
  

$$3x(x+4) - 1(x+4)$$
  

$$(x+4)(3x-1)$$

19. Solve:  $x^3 - 5x^2 + 16x = 80$ . Which of the following is the factored form of the equation and the solution set?

- A.  $(x+5)(x^2+16); \{5, 4i, -4i\}$   
 B.  $(x-5)(x^2+16) = 0; \{5, 4, -4\}$   
 C.  $(x-5)(x^2+16); \{5, 4i, -4i\}$   
 D.  $(x-5)(x+16)^2; \{5, -16\}$

$$x^3 - 5x^2 + 16x - 80 = 0$$
  

$$x^2(x-5) + 16(x-5) = 0$$
  

$$(x-5)(x^2+16) = 0$$
  

$$5, \pm 4i$$

20-22: **SOLVE** each polynomial over the set of complex numbers by **FACTORING**. (GCF, factor by grouping...etc) (Provide exact solutions only, no decimal answers.) Hint: Use the calculator!

20.  $x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

\

factored form:

$(x-4)(x+2) = 0$

solutions:

$x = 4, -2$

21.  $x^3 + 4x^2 - 3x - 12 = 12$

$$x^3 + 4x^2 - 3x - 12 = 0$$
  

$$x^2(x+4) - 3(x+4) = 0$$
  

$$(x+4)(x^2-3) = 0$$

factored form:

$(x+4)(x^2-3) = 0$

solutions:

$-4, \pm\sqrt{3}$

22.  $2x^4 - 200x^2 = 0$

$$2x^2(x^2 - 100) = 0$$
  

$$2x^2(x+10)(x-10) = 0$$

factored form:

$2x^2(x+10)(x-10) = 0$

solutions:

$x = 0, 0, -10, 10$

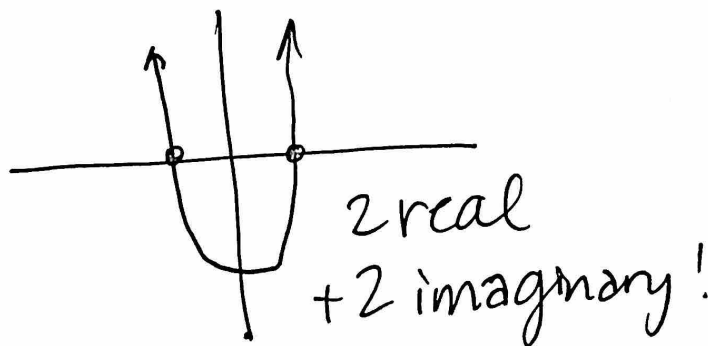
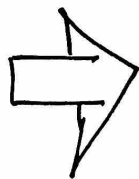
**Learning Target 6** I can find the zeros of a higher degree polynomial function over the set of complex numbers using the

process of depressing a polynomial.

Circle the best solution.

23. Which describes the number and type of roots of the equation  $m^4 - 16 = 0$ ?

- A. 2 real roots and 2 imaginary roots
- B. 4 real roots
- C. 4 imaginary roots
- D. 3 real roots with one double root



24. Given the function  $f(x) = x^3 + 8x^2 + 22x + 21$  solve over the set of complex numbers.

Hint: Find all the roots.

(Provide exact solutions only, no decimal solutions.)

$$\begin{array}{r|rrrr} -3 & 1 & 8 & 22 & 21 \\ & \downarrow & -3 & -15 & -21 \\ \hline & 1 & 5 & 7 & \end{array} \quad \text{calc. } -3$$

$$(x+3)(x^2+5x+7) = 0$$

$$x = \frac{-5 \pm \sqrt{25-4(7)}}{2}$$
$$\frac{-5 \pm \sqrt{-3}}{2}$$

$$x = -3, \frac{-5 \pm i\sqrt{3}}{2}$$

redo

$$f(x) = x^4 + x^3 + 2x^2 + 4x - 8$$

25. Given the function  $f(x) = x^3 + 5x^2 - 2x - 10$ , solve over the set of complex numbers.

Hint: Find all the roots.

(Provide exact solutions only, no decimal solutions.)

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 2 & 4 & -8 \\ & \downarrow & -2 & 2 & -8 & 8 \\ \hline 1 & 1 & -1 & 4 & -4 & 0 \\ & \downarrow & 1 & 0 & 4 & \\ \hline & 1 & 0 & 4 & 0 & \end{array} \quad \text{calc } -2 \& 1$$

$$(x+2)(x-1)(x^2+4) = 0$$

$$x = -2, 1, \pm 2i$$