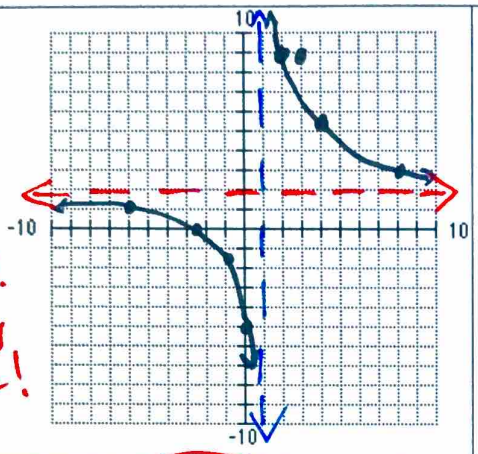
Algebra 2 Unit 4 Test Review
Rational Expressions, Equations & Functions

Name: Master E
Date: _____ **Block:** 2

Learning Target 1: I CAN graph a rational function, identify its transformations, components (domain, range, zeros, intercepts), and its restrictions (vertical, horizontal, and oblique asymptotes, and points of discontinuity).

1-2 : Graph each of the following functions and provide the indicated information below. Plot at least two points on each branch of the graph, and sketch asymptotes as dashed lines. Use interval notation for the domain and range.

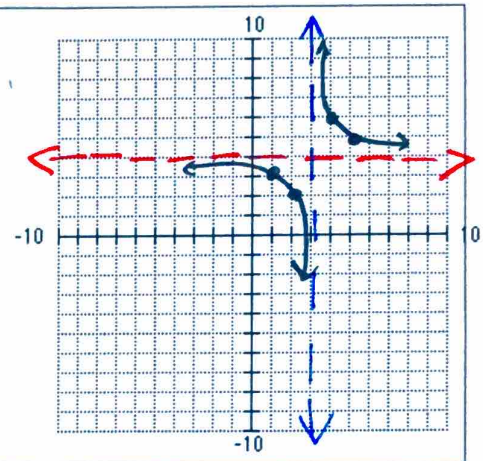
1. $f(x) = \frac{2x+5}{x-1}$



Vertical is always x=!
Horizontal is always y=!

vertical asymptote(s):	horizontal asymptote:
$x=1$	$y=\frac{2}{1}$ $y=2$
domain:	**range:**
$(-\infty,1)\cup(1,\infty)$	$(-\infty,2)\cup(2,\infty)$

2. $f(x) = \frac{2}{x-3}+4$



vertical asymptote(s):	horizontal asymptote:
$x=3$	$y=4$
domain:	**range:**
$(-\infty,3)\cup(3,\infty)$	$(-\infty,4)\cup(4,\infty)$

Always check EVERYTHING ON DESMOS including typing in your asymptotes!

3-6 : ***Write the equation of the function when*** $f(x)=\frac{2}{x}$ ***is translated and ...***

3. ...has horizontal asymptote y = 3 and vertical asymptote x = -5. $f(x) = \frac{2}{x+5}+3$

4. ...has horizontal asymptote y = -6 and vertical asymptote x = -3. $f(x) = \frac{2}{x+3}-6$

5. ...shifts the graph up 8 and right 2. $f(x) = \frac{2}{x-2}+8$

6. ...reflects the graph over the x-axis. $f(x) = -\frac{2}{x}$

Only reciprocal functions can be translated from the parent function ... $f(x)=\frac{a}{x-h}+k$

Learning Target 2: I CAN model and solve real world problems by using direct, inverse, and joint variation or a combination of direct and inverse variation.

7-9 : The Volume *V* of wood in a tree varies jointly as the height (h) and the square of the girth (g). If the volume of a tree is 144 cubic meters when the height is 20 meters and the girth is 1.5 meters, what is the height of a tree when the volume is 1000 cubic meters and a girth of 2 meters? SHOW YOUR WORK!

7. Find the value of the constant of variation (k).

$k = 3.2$

$V = khg^2$

$144 = k(20)(1.5)^2$

$144 = k(45)$

$k = 3.2$

8. Write the equation that relates the Volume to the height and girth.

$V = 3.2hg^2$

9. Find the height of a tree if the volume is 1000 m^3 and has a girth of 1.5 m.

78.125 meters

$V = 3.2hg^2$

$1000 = 3.2(h)(2)^2$

$\frac{1000}{12.8} = \frac{12.8h}{12.8}$

10-11 : SHOW YOUR WORK to determine if the given problem represents direct, inverse, joint or no variation.

10. The equation $\frac{y}{x} = \frac{8}{11}$ represents D variation?

$11 \cdot y = 8 \cdot x$

$\frac{11y}{11} = \frac{8x}{11}$

$y = \frac{8}{11}x$

Direct

11. The table represents D variation.

x	y
3	2
6	4
10	20/3
12	8

$x \cdot y \neq k$

$\frac{y}{x} = k$

$\frac{y}{x} = \frac{2}{3}$

$3y = 2x$ $y = \frac{2}{3}x$

Direct

12. Suppose y varies jointly as x and z. If y = 48 when x = 2 and z = 8, find y when x = 8 and z = 4.

$y = kxz$

$48 = k(2)(8)$

$48 = 16k$

$3 = k$

$y = 3xz$

$y = 3(8)(4)$

$y = 96$

13. The volume of a pyramid *V*, varies jointly as its height *h* and the area of its base A_b. Write the equation that represents this variation.

$V = \frac{k}{hA}$

If it was inverse, $x \cdot y$ would = a constant

example

$x \cdot y = 12$

$\therefore y = \frac{12}{x}$

x	y
1	12
2	6
3	4
6	2

Learning Target 3: I CAN add, subtract, multiply, divide, and simplify rational expressions.

14-19 : *Simplify each expression completely. Neatly SHOW YOUR WORK and then circle your answer.*

14. $\dfrac{\dfrac{x^2-9x-22}{x^2+5x-24}}{\dfrac{x+2}{x-3}}$

$\dfrac{x^2-9x-22}{x^2+5x-24} \cdot \dfrac{x-3}{x+2}$

$\dfrac{(x-11)(x+2)(x-3)}{(x+8)(x-3)(x+2)}$

$\boxed{\dfrac{x-11}{x+8}}$

15. $\dfrac{x+7}{2-x} - \dfrac{x-5}{x-2}$

$\dfrac{x+7}{2-x} + \dfrac{-x+5}{x-2}$ (-1) (-1)

$\dfrac{x+7}{2-x} + \dfrac{x-5}{2-x}$

$\dfrac{x+7+x-5}{2-x}$

$\boxed{\dfrac{2x+2}{2-x}}$

16. $\dfrac{10}{x^2-9} + \dfrac{5}{x+3}$

$\dfrac{10}{(x+3)(x-3)} + \dfrac{5}{(x+3)}\dfrac{(x-3)}{(x-3)}$

$\dfrac{10+5x-15}{(x+3)(x-3)}$

$\boxed{\dfrac{5x-5}{(x+3)(x-3)}}$

17. $\dfrac{x^2+4x-5}{x^2+x-2} \cdot \dfrac{3x+6}{x+5}$

$\dfrac{(x+5)(x-1)}{(x+2)(x-1)} \cdot \dfrac{3(x+2)}{(x+5)}$

$\boxed{3}$

18. $\dfrac{x-4y}{10} \div \dfrac{x^2-16y^2}{12}$

$\dfrac{x-4y}{10} \cdot \dfrac{12}{x^2-16y^2}$

$\dfrac{(x-4y)}{2\cdot 5} \cdot \dfrac{2\cdot 6}{(x+4y)(x-4y)}$

$\boxed{\dfrac{6}{5(x+4y)}}$

19. $\dfrac{x-2}{3x+15} - \dfrac{x+4}{x+5}$

$\dfrac{x-2}{3(x+5)} - \dfrac{x+4}{(x+5)}$ (3)

$\dfrac{x-2-3x-12}{3(x+5)}$

$\boxed{\dfrac{-2x-14}{3(x+5)}}$ or $\boxed{\dfrac{-2(x+7)}{3(x+5)}}$

★ To add or subtract fractions, you have to get like denominators!

○ To multiply or divide fractions, you have to completely factor first & then reduce like factors. Always check for the GCF first!

Learning Target 4: I can solve equations containing rational expressions algebraically and check their solutions graphically.

20-23: Solve each of the following equations. Show all work, circle your answers and be sure to state restricted values.

20. $\frac{1}{n} = \frac{1}{4n^2} + \frac{n+3}{4n^2}$ LCD $4n^2$

$4n^2\left(\frac{1}{n}\right) = 4n^2\left(\frac{1}{4n^2}\right) + 4n^2\left(\frac{n+3}{4n^2}\right)$

$4n = 1 + n + 3$

$4n = n + 4$

$3n = 4$

$n = \frac{4}{3}$

21. $\frac{2x-4}{x-2} = \frac{4}{x-4}$

$(2x-4)(x-4) = 4(x-2)$

$2x^2 - 8x - 4x + 16 = 4x - 8$

$2x^2 - 12x + 16 = 4x - 8$

$2x^2 - 16x + 24 = 0$

$2(x^2 - 8x + 12) = 0$

$2(x-6)(x-2) = 0$

$x = 6, \; x = 2$

$x \neq 2$ extraneous

F.A. $x = 6$

22. $\frac{1}{x-2} + \frac{1}{x+3} = \frac{5}{x^2+x-6}$ LCD $(x+3)(x-2)$

$(x+3)(x-2)\left(\frac{1}{x-2}\right) + (x+3)(x-2)\left(\frac{1}{x+3}\right) = \frac{5}{(x+3)(x-2)}(x+3)(x-2)$

$x + 3 + x - 2 = 5$

$2x + 1 = 5$

$2x = 4$

$x = 2$

$x \neq 2$ extraneous

NO SOLUTION

23. $\frac{r+1}{r} + \frac{r-4}{2r} = \frac{3r^2-2r-5}{5r}$

LCD $10r$

$10r\left(\frac{r+1}{r}\right) + 10r\left(\frac{r-4}{2r}\right) = \left(\frac{3r^2-2r-5}{5r}\right)10r$

$10(r+1) + 5(r-4) = 2(3r^2 - 2r - 5)$

$10r + 10 + 5r - 20 = 6r^2 - 4r - 10$

$15r - 10 = 6r^2 - 4r - 10$

$6r^2 - 19r = 0$

$r(6r - 19) = 0$

$r = 0 \quad 6r - 19 = 0$

$6r = 19$

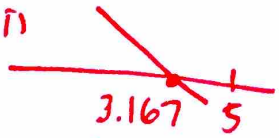
$r \neq 0$ extraneous

$r = \frac{19}{6}$

To check your work, go to Desmos & use x for every variable... for example, move right side over

$\frac{(x+1)}{r} + \frac{(x-4)}{2r} - \frac{(3x^2-2x-5)}{5r}$

& look where it crosses the x-axis, which is at 3.167



$\frac{19}{6}$ does = 3.167!