# 9-1 Multiplying & Dividing Rational Expressions

Name ___Master E___
Date _____ Block ___

## REMEMBER THE POWER FORMULAS!

| | |
|---|---|
| When multiplying monomials, you **ADD** exponents | $x^2 \cdot x^3 = x^{3+2} = x^5$ |
| When dividing monomials, you **SUBTRACT** exponents | $\dfrac{10x^8}{2x^6} = 5x^{8-6} = 5x^2$ |
| When raising a power to a power, you **MULTIPLY** exponents | $(x^3)^4 = x^{3 \cdot 4} = x^{12}$ |

## HOW TO SIMPLIFY A RATIONAL EXPRESSION:

1) Factor the numerator and denominator completely (don't make up your own rules!)
2) Divide out any factors that are common to both the numerator and denominator.
3) Simplify (*A rational expression is simplified if its numerator and denominator have no common factors other than 1*).

**EXAMPLE:** $\dfrac{x^2 - 5x - 6}{x^2 - 1} = \dfrac{(x-6)(x+1)}{(x+1)(x-1)} = \boxed{\dfrac{x-6}{x-1}}$

1. $\dfrac{4x^2}{2x^2 + 3x}$ $\quad \dfrac{4x^2}{x(2x+3)} = \boxed{\dfrac{4x}{2x+3}}$

2. $\dfrac{x^2 - 2x - 15}{x^2 - 4x - 5}$ $\quad \dfrac{(x-5)(x+3)}{(x-5)(x+1)}$ $\quad \boxed{\dfrac{x+3}{x+1}}$

3. $\dfrac{x^2 - 16}{x^2 + x - 12}$ $\quad \dfrac{(x+4)(x-4)}{(x+4)(x-3)}$ $\quad \boxed{\dfrac{x-4}{x-3}}$

4. $\dfrac{3x^2 - 5x - 2}{x^2 - 4}$ $\quad \dfrac{(3x+1)(x-2)}{(x+2)(x-2)} = \boxed{\dfrac{3x+1}{x+2}}$

5. $\dfrac{2x+6}{x^2 - 6x + 9}$ $\quad \dfrac{2(x+3)}{(x-3)^2}$

6. $\dfrac{x^2 - 25}{x^3 - 125}$ $\quad \dfrac{(x+5)(x-5)}{(x-5)(x^2+5x+25)}$ $\quad \boxed{\dfrac{x+5}{x^2+5x+25}}$

## HOW TO MULTIPLY RATIONAL EXPRESSIONS WITH MONOMIALS:

1) First, multiply the numerators and denominators
2) Reduce the integers and use your power rules to simplify the variable expression.

**EXAMPLE:** $\dfrac{6x^2 y^3}{2x^2 y^2} \cdot \dfrac{10x^3 y^4}{18y^2} = \dfrac{60x^5 y^7}{36x^2 y^4} = \dfrac{12 \cdot 5 x^{5-2} y^{7-4}}{12 \cdot 3} = \dfrac{5x^3 y^3}{3}$

7. $\dfrac{12x^2 y}{5y^2} \cdot \dfrac{2xy}{3x^2}$ $\quad \boxed{\dfrac{8x}{5}}$

8. $\dfrac{4y^2}{9x} \cdot \dfrac{27}{16xy^2}$ $\quad \boxed{\dfrac{3}{4x^2}}$

## HOW TO MULTIPLY RATIONAL EXPRESSIONS WITH POLYNOMIALS:

1) Factor each polynomial in the numerator and the denominator
2) Multiply all factors together in factored form.
3) Reduce like factors. $\dfrac{x+1}{x+1}=1$ ; $\dfrac{x+1}{1+x}=1$ ; $\dfrac{x-1}{1-x}=-1$ ; $\dfrac{x+1}{x-1}$ is simplified!
4) Multiply what is left together. You may leave it in factored form if you wish.

**EXAMPLE:** $\dfrac{3x-27x^3}{3x^2-2x-1} \cdot \dfrac{3x^2-4x+1}{3x} = \dfrac{(3x-27x^3)(3x^2-4x+1)}{(3x^2-2x-1)(3x)} = \dfrac{(3x)(1-9x^2)(3x-1)(x-1)}{(3x+1)(x-1)(3x)} =$

$\dfrac{(3x)(1+3x)(1-3x)(3x-1)(x-1)}{(3x+1)(x-1)(3x)} = (1-3x)(3x-1) = -1(3x-1)(3x-1) = -(3x-1)^2$

9. $\dfrac{x^2-2x}{x^2+2x+1} \cdot \dfrac{x^2+4x+3}{x^2+3x}$

$\dfrac{x(x-2)(x+3)(x+1)}{(x+1)(x+1)x(x+3)}$

$\boxed{\dfrac{x-2}{x+1}}$

10. $\dfrac{a-y}{w+n} \cdot \dfrac{w^2-n^2}{y-a}$

$\dfrac{(a-y)(w+n)(w-n)}{(w+n)-1(a-y)}$

$\boxed{-w+n}$ or $\boxed{n-w}$

11. $\dfrac{x^2-5x-24}{6x+2x^2} \cdot \dfrac{5x^2}{8-x}$

$\dfrac{(x-8)(x+3)\,5x^2}{2x(3+x)(x+8)(-1)}$

$\boxed{\dfrac{-5x}{2}}$

## HOW TO DIVIDE RATIONAL EXPRESSIONS:

1) Multiply by the reciprocal of the 2$^{nd}$ expression (change division to multiplication and flip the fraction!)
2) Follow the same steps for multiplication.

**EXAMPLE:** $\dfrac{6x^2+7x-3}{x-1} \div (3x-1) = \dfrac{6x^2+7x-3}{x-1} \cdot \dfrac{1}{3x-1} = \dfrac{(3x-1)(2x+3)}{(x-1)(3x-1)} = \dfrac{2x+3}{x-1}$

12. $\dfrac{5x^5}{8} \div \dfrac{15x^2}{12}$

$\dfrac{5x^5}{8} \cdot \dfrac{12}{15x^2}$

$\boxed{\dfrac{x^3}{2}}$

13. $\dfrac{48x^2}{y} \div \dfrac{36xy^2}{5}$

$\dfrac{48x^2}{y} \cdot \dfrac{5}{36xy^2}$

$\boxed{\dfrac{20x}{3y^3}}$

14. $(x^2+10x-24) \div \dfrac{x^2-144}{3x-36}$

$(x+12)(x-2) \cdot \dfrac{3(x-12)}{(x+12)(x-12)}$

$\boxed{3(x-2)}$

15. $\dfrac{2x^3-12x^2}{x^2-4x-12} \div \dfrac{8x^3+24x^2}{x^2+9x+18}$

$\dfrac{2x^2(x-6)}{(x-6)(x+2)} \cdot \dfrac{(x+6)(x+3)}{8x^2(x+3)}$

$\boxed{\dfrac{(x+6)}{4(x+2)}}$

16. $\dfrac{\frac{x^2-9}{4}}{\frac{3-x}{8}}$

$\dfrac{(x+3)(x-3)}{4} \cdot \dfrac{8}{(3-x)}$

$\boxed{-2(x+3)}$

17. $\dfrac{\frac{y^4-81}{xy+4y+3x+12}}{\frac{y^2+9}{}}$  $y(x+4)+3(x+4)$

$\dfrac{(y^2+9)(y^2-9)}{(y+3)(x+4)} \cdot \dfrac{1}{(y^2+9)}$

$\dfrac{(y+3)(y-3)}{(y+3)(x+4)} = \boxed{\dfrac{y-3}{x+4}}$