

Investigating Oblique/Slant Asymptotes and Holes

Name _____

Date _____ Block _____

1. $f(x) = \frac{2x^2 - 4x}{x+1} = \frac{2x(x-2)}{x+1}$

- a. What are the equations of the horizontal and vertical asymptotes of the graph of this function?

Vertical: $x = -1$

horizontal: none (num. deg. > denom. deg.)

- b. What are the zeros/x-intercepts of the graph?

$(0,0)$ & $(2,0)$

- c. What is the y-intercept of the graph?

$(0,0)$

- d. Graph the function on a graphing calculator and explore the table of values. Does anything about the graph appear unusual compared to the graphs of other rational functions you have seen?

$$x+1 \overline{) 2x^2 - 4x}$$

$$\underline{2x^2 + 2x}$$

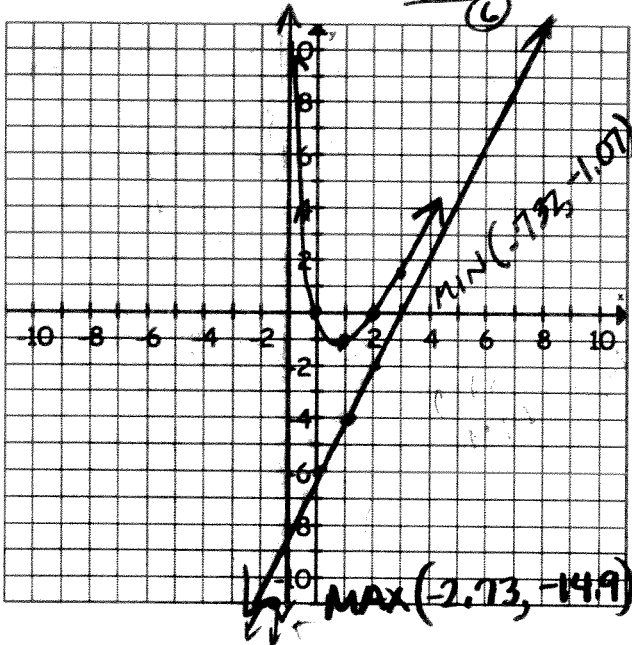
$$-6x + 0$$

$$\underline{-6x + 6}$$

$$-6$$

$y = 2x - 6$

- e. Graph the function on the grid below.



- f. What are the domain and range of the function?

$D: (-\infty, -1) \cup (-1, \infty)$

$R: (-\infty, -14.9] \cup [-1.07, \infty)$

2. $f(x) = \frac{(x+3)(x^2 + 3x - 10)}{x-2} = \frac{(x+3)(x+5)(x-2)}{(x-2)}$

- a. What are the equations of the horizontal and vertical asymptotes of the graph of this function?

$x = 2$?

no horizontal asymptote b/c x^3 is $> x$

- b. What are the zeros/x-intercepts of the graph?

$(-3,0)$ $(5,0)$ $(2,0)$?

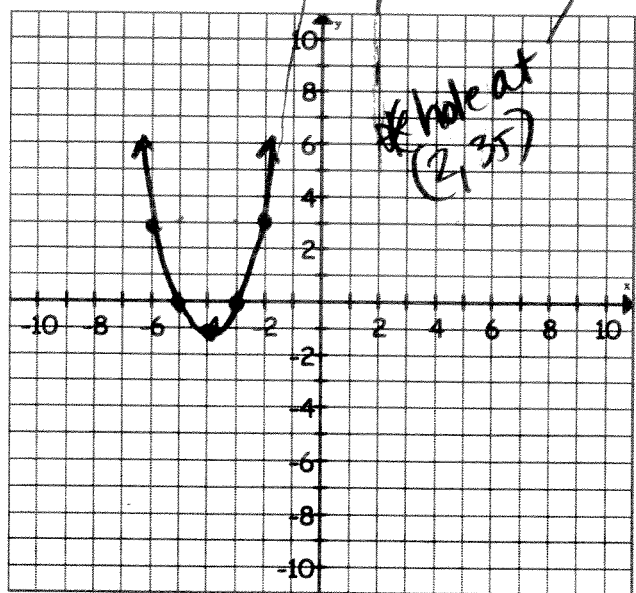
- c. What is the y-intercept of the graph?

$\frac{3(-10)}{-2} = 15$ $(0,15)$ ✓

- d. Graph the function on a graphing calculator and explore the table of values. Does anything about the graph appear unusual compared to the graphs of other rational functions you have seen?

$x = 2$ is not an asymptote?
It is a parabola.

- e. Graph the function on the grid below.



- f. What are the domain and range of the function?

$D: (-\infty, 2) \cup (2, \infty)$

$R: [-1, \infty)$

Graphs of Rational Functions: Oblique/Slant Asymptotes and Holes

Find all vertical and slant asymptotes of the graph of each of the following.

1. $f(x) = \frac{x^2}{x-1} \cdot \frac{x+1}{x-1} = \frac{x^2+x+1}{x^2-2x+1}$
 $X=1$
 $Y=X+1$

2. $f(x) = \frac{x^2+1}{x}$
 $X=0$
 $Y=X$

3. $f(x) = \frac{x^2+x-1}{x-1} \cdot \frac{x+2}{x+2} = \frac{x^2+3x-1}{x^2-x-2}$
 $X=1$
 $Y=X+2$

4. $f(x) = \frac{2x^2-5}{x+3} \cdot \frac{x+3}{x+3} = \frac{2x^2+6x-5}{x^2+6x+9}$
 $X=-3$
 $Y=2X-6$

5. $f(x) = \frac{x^3}{x^2-4} = \frac{x^3}{(x+2)(x-2)}$
 $X=2$
 $X=-2$
 $Y=X$

6. $f(x) = \frac{9-x^2}{2+x} \cdot \frac{x-2}{x-2} = \frac{-x^2+9}{x^2-x-2}$
 $X=-2$
 $Y=-X+2$

State the coordinates of all holes in the graph of each of the following rational functions.

7. $f(x) = \frac{x-3}{3-x} = \frac{X-3}{-1(X-3)}$
 $(3, -1)$

8. $f(x) = \frac{x+5}{x+5}$
 $(-5, 1)$

9. $f(x) = \frac{2x+6}{x+3} = \frac{2(X+3)}{X+3}$
 $(-3, 2)$

10. $f(x) = \frac{x^2-4}{x+2} = \frac{(X+2)(X-2)}{X+2}$
 $(-2, -4)$

11. $f(x) = \frac{x^2-3x-10}{x+2} = \frac{(X-5)(X+2)}{X+2}$
 $(-2, -7)$

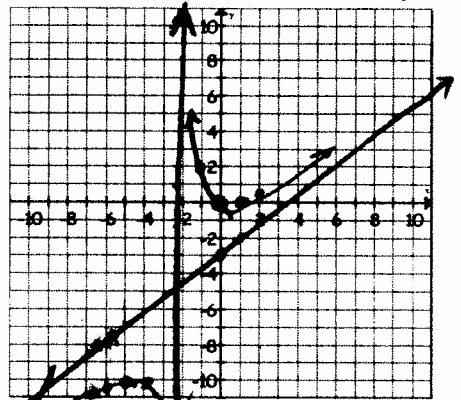
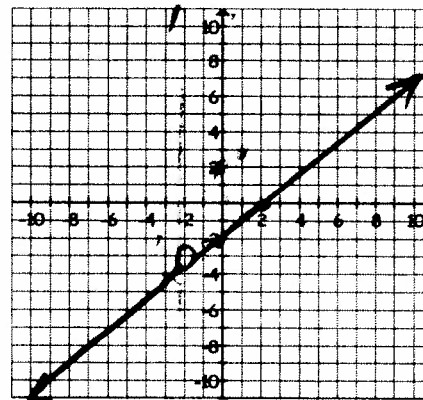
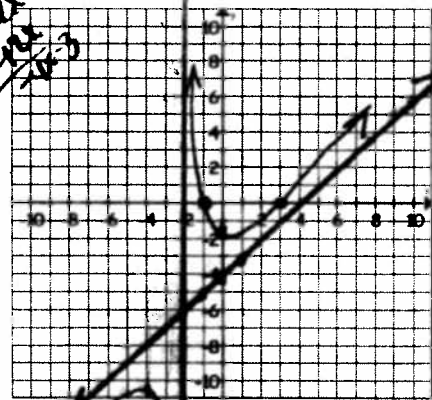
12. $f(x) = \frac{x^3+x^2-x-1}{x+1} = \frac{x^2(x+1)-1(x+1)}{(x+1)}$
 $(-1, 0)$

Graph each rational function and find the following, as applicable, for each function: y-intercept, zeros, equations of all vertical, horizontal, and slant asymptotes, coordinates of holes, and the domain and range.

13. $f(x) = \frac{x^2-2x-3}{x+2} = \frac{(X-3)(X+1)}{X+2}$

14. $f(x) = \frac{x^2-4}{x+2} = \frac{(X+2)(X-2)}{X+2}$

15. $e(x) = \frac{x^2-x}{x+2} = \frac{X(X-1)}{X+2}$



y-intercept: $(0, -\frac{3}{2})$
 zeros: $(3, 0), (-1, 0)$
 asymptotes: $X=-2$
 $Y=X-4$
 holes: none
 domain: $(-\infty, -2) \cup (-2, \infty)$
 range: $(-\infty, -10.5] \cup (-1.5, \infty)$
 max: $(-4.24, -10.5)$
 min: $(0, 24, -1.53)$

y-intercept: $(0, -2)$
 zeros: $(2, 0)$
 asymptotes: $X=-2$
 slant: N/A
 holes: $(-2, -4)$
 domain: $(-\infty, -2) \cup (-2, \infty)$
 range: $(-\infty, -4) \cup (-4, \infty)$

y-intercept: $(0, 0)$
 zeros: $(0, 0), (1, 0)$
 asymptotes: $X=-2$
 $Y=X-3$
 holes: none
 domain: $(-\infty, -2) \cup (-2, \infty)$
 range: $(-\infty, -9.9] \cup [-0.1, \infty)$
 max: $(-4.45, -9.9)$
 min: $(0.45, -0.1)$