Analyzing Graphs of Polynomial Functions

Name Master E

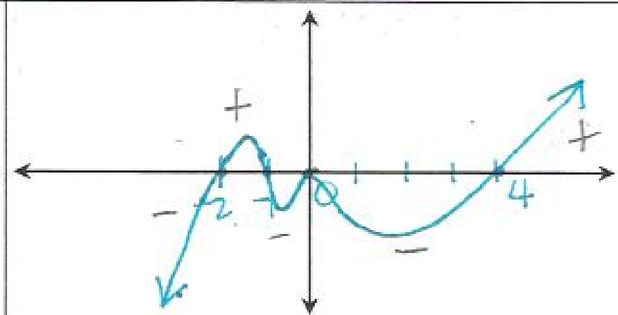
Date ____________ Block ______

For each polynomial, state the requested information and sketch the graph.
DO NOT USE YOUR CALCULATOR WHEN SKETCHING THE GRAPH!

1. $f(x) = x(x^2 + 2x)(x^2 - 3x - 4) = x^2(x+2)(x-4)(x+1)$

degree: 5
even or odd: ODD
max turns: 4
max x-int: 5
zeros at: 0, 0, −2, 4, −1
tangent at: 0

terrace at: none
as $x \to -\infty$: $f(x) \to -\infty$
as $x \to +\infty$: $f(x) \to +\infty$
Estimate the x-coordinates for:
local max(s): betw −1 & −2, 0
local min(s): betw. −1 & 0; betw. 0 & 4



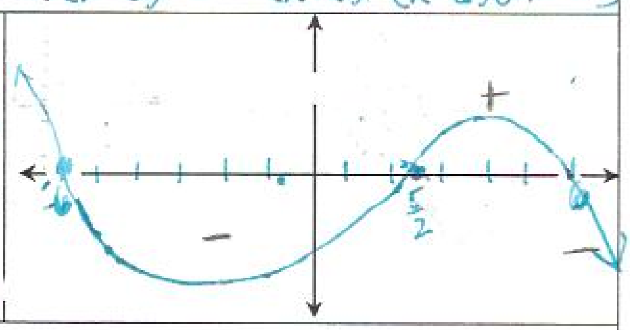
Sign Change Chart:

	x^2	$x+2$	$x-4$	$x+1$	$f(x)$
−3	+	−	−	−	−
−1.5	+	+	−	−	+
−.5	+	+	−	+	−
2	+	+	−	+	−
5	+	+	+	+	+

2. $f(x) = -(x^2 - 36)(2x - 5)(x + 6)^2 = -(x+6)(x-6)(2x-5)(x+6)^2 = -(x+6)^3(x-6)(2x-5)$

degree: 5
even or odd: ODD
max turns: 4
max x-int: 5
zeros at: −6, −6, −6, 6, $\frac{5}{2}$ (2.5)
tangent at: none

terrace at: −6
as $x \to -\infty$: $f(x) \to +\infty$
as $x \to +\infty$: $f(x) \to -\infty$
Estimate the x-coordinates for:
local max(s): betw. −6 & $\frac{5}{2}$
local min(s): betw. $\frac{5}{2}$ & 6



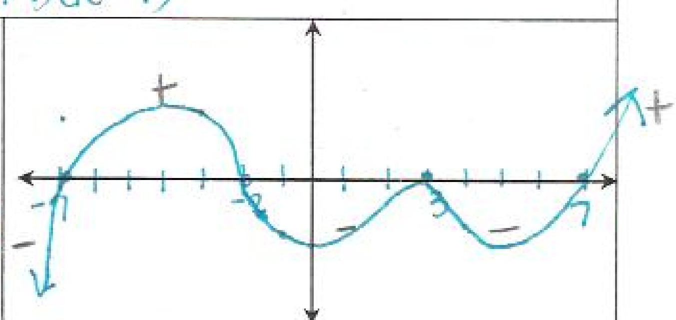
Sign Change Chart:

	−	$(x+6)^3$	$(x-6)$	$(2x-5)$	$f(x)$
−7	−	−	−	−	+
0	−	+	−	−	−
4	−	+	−	+	+
7	−	+	+	+	−

3. $f(x) = (x + 2)^3(x - 3)^2(x^2 - 49) = (x+2)^3(x-3)^2(x+7)(x-7)$

degree: 7
even or odd: ODD
max turns: 6
max x-int: 7
zeros at: −2, −2, −2, 3, 3, −7, 7
tangent at: 3

terrace at: −2
as $x \to -\infty$: $f(x) \to -\infty$
as $x \to +\infty$: $f(x) \to +\infty$
Estimate the x-coordinates for:
local max(s): btw −7 & −2, 3
local min(s): btw −2 & 3; btw 3 & 7



Sign Change Chart:

Test Point	$(x+2)^3$	$(x-3)^2$	$(x+7)$	$(x-7)$	$f(x)$
−8	−	+	−	−	−
−5	−	+	+	−	+
0	+	+	+	−	−
4	+	+	+	−	−
8	+	+	+	+	+

4. $f(x) = (x+4)^4(x+1)(x-3)^3$

degree: 8

even or odd: EVEN

max turns: 7

max x-int: 8

zeros at: -4, -4, -4, -4, -1, 3, 3, 3

tangent at: -4

terrace at: 3

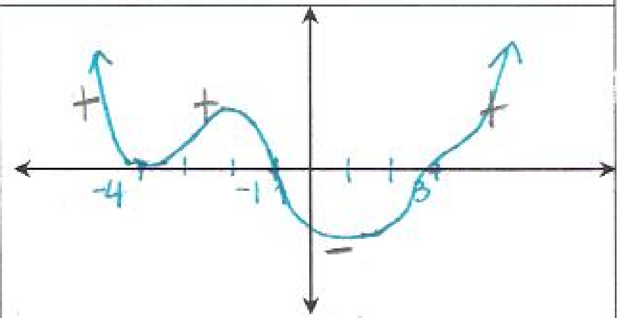
as $x \to -\infty$: $f(x) \to +\infty$

as $x \to +\infty$: $f(x) \to +\infty$

Estimate the x-coordinates for:

local max(s): b/w -4 & -1

local min(s): -4, b/w -1 & 3



Sign Change Chart:

Test Point	$(x+4)^4$	$(x+1)$	$(x-3)^3$	$f(x)$
-5	+	−	−	+
-2	+	−	−	+
0	+	+	−	−
4	+	+	+	+

5. $f(x) = -x^2(x+3)(x-5)(x^2-9)^2 = -x^2(x+3)(x-5)(x+3)^2(x-3)^2 = -x^2(x+3)^3(x-5)(x-3)^2$

degree: 8

even or odd: EVEN

max turns: 7

max x-int: 8

zeros at: 0, 0, -3, -3, -3, 5, 3, 3

tangent at: 0 & 3

terrace at: -3

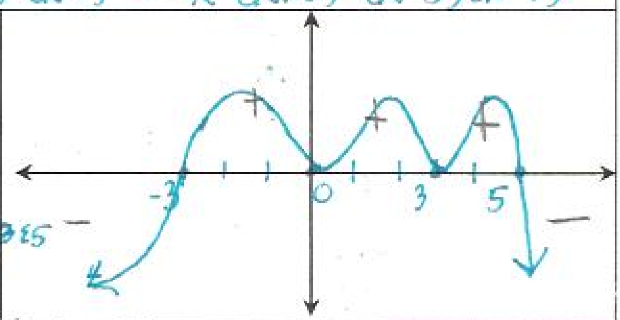
as $x \to -\infty$: $f(x) \to -\infty$

as $x \to +\infty$: $f(x) \to -\infty$

Estimate the x-coordinates for:

local max(s): b/w -3 & 0, 0 & 3, 3 & 5

local min(s): 0, 3



Sign Change Chart:

	$-x^2$	$(x+3)^3$	$(x-5)$	$(x-3)^2$	$f(x)$
-4	−	−	−	+	−
-1	−	+	−	+	+
1	−	+	−	+	+
4	−	+	−	+	+
6	−	+	+	+	−

6. $f(x) = -x(x^3+2x^2)(x^2+2x-15) = -x \cdot x^2(x+2)(x+5)(x-3) = -x^3(x+2)(x+5)(x-3)$

degree: 6

even or odd: EVEN

max turns: 5

max x-int: 6

zeros at: 0, 0, 0, -2, -5, 3

tangent at: none

terrace at: 0

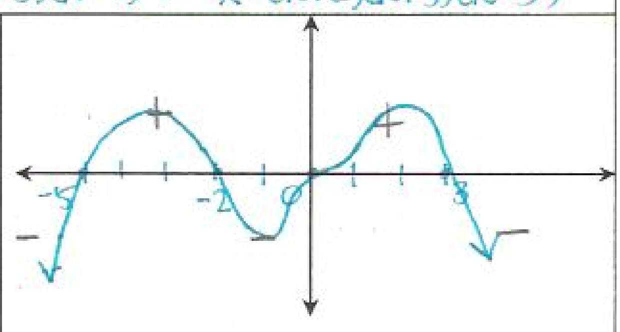
as $x \to -\infty$: $f(x) \to -\infty$

as $x \to +\infty$: $f(x) \to -\infty$

Estimate the x-coordinates for:

local max(s): b/w -5 & -2, 0 & 3

local min(s): b/w -2 & 0



Sign Change Chart:

Test Point	$-x^3$	$(x+2)$	$(x+5)$	$(x-3)$	$f(x)$
-6	+	−	−	−	−
-4	+	−	+	−	+
-1	+	+	+	−	−
2	−	+	+	−	+
4	−	+	+	+	−

7. $f(x) = (x^2 - 4)(2x - 1)(x + 5)^2$ $= (x+2)(x-2)(2x-1)(x+5)^2$

degree: 5

even or odd: ODD

max turns: 4

max x-int: 5

zeros at: $-2, 2, \frac{1}{2}, -5, -5$

tangent at: -5

terrace at: none

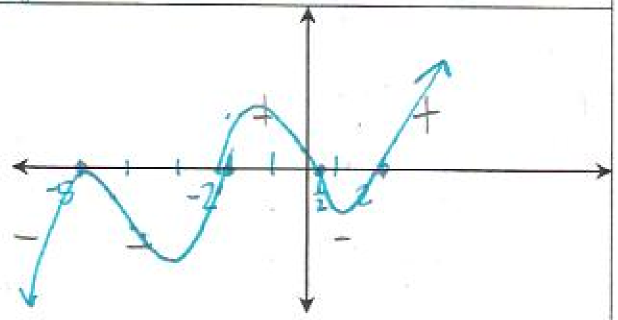
as $x \to -\infty$: $f(x) \to -\infty$

as $x \to +\infty$: $f(x) \to +\infty$

Estimate the x-coordinates for:

local max(s): -5, btw -2 & $\frac{1}{2}$

local min(s): btw -5 & -2, btw $\frac{1}{2}$ & 2



Sign Change Chart:

Test Point	$x+2$	$x-2$	$2x-1$	$(x+5)^2$	$f(x)$
-6	−	−	−	+	−
-3	−	−	−	+	−
-1	+	−	−	+	+
1	+	−	+	+	−
3	+	+	+	+	+

8. $f(x) = (x + 1)^3(x + 4)^2(x^2 - 25)$ $= (x+1)^3(x+4)^2(x+5)(x-5)$

degree: 7

even or odd: ODD

max turns: 6

max x-int: 7

zeros at: $-1, -1, -1, -4, -4, -5, 5$

tangent at: -4

terrace at: -1

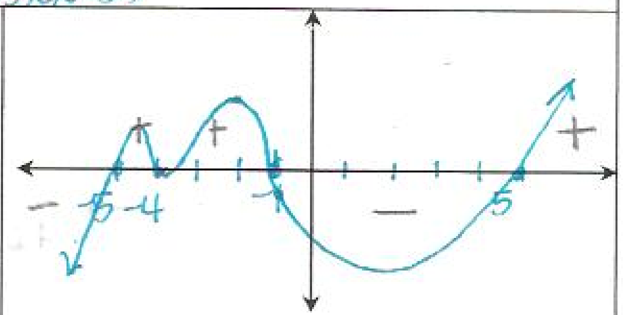
as $x \to -\infty$: $f(x) \to -\infty$

as $x \to +\infty$: $f(x) \to +\infty$

Estimate the x-coordinates for:

local max(s): btw -5 & -4, -4 & -1

local min(s): -4, btw -1 & 5



Sign Change Chart:

	$(x+1)^3$	$(x+4)^2$	$(x+5)$	$(x-5)$	$f(x)$
-6	−	+	−	−	−
-4.5	−	+	+	−	+
-3	−	+	+	−	+
2	+	+	+	−	−
6	+	+	+	+	+

9. $f(x) = -(x + 2)^4(x + 5)(x - 6)^3$

degree: 8

even or odd: EVEN

max turns: 7

max x-int: 8

zeros at: $-2, -2, -2, -2, -5, 6, 6, 6$

tangent at: -2

terrace at: 6

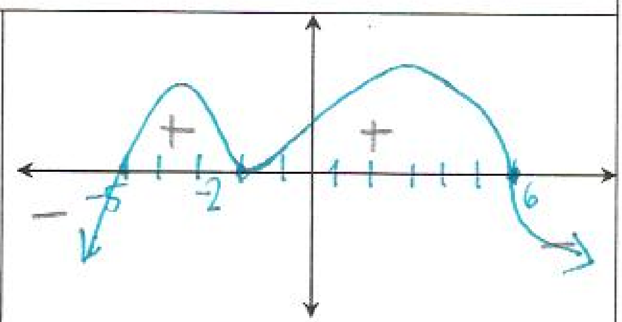
as $x \to -\infty$: $f(x) \to -\infty$

as $x \to +\infty$: $f(x) \to -\infty$

Estimate the x-coordinates for:

local max(s): btw -5 & -2, btw -2 & 6

local min(s): -2



Sign Change Chart:

Test Point	$-(x+2)^4$	$(x+5)$	$(x-6)^3$	$f(x)$
-6	−	−	−	−
-4	−	+	−	+
1	−	+	−	+
7	−	+	+	−

10. $f(x) = x^3(x-1)(x-5)(x^2-16)^2 = x^3(x-1)(x-5)(x+4)^2(x-4)^2$

degree: 9

even or odd: ODD

max turns: 8

max x-int: 9

zeros at: 0,0,0,1,5,-4,-4,4,4

tangent at: -4, 4

terrace at: 0

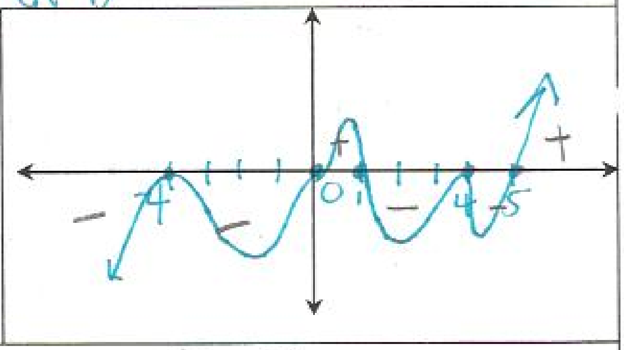
as $x \to -\infty$: $f(x) \to -\infty$

as $x \to +\infty$: $f(x) \to +\infty$

Estimate the x-coordinates for:

local max(s): -4, btw 0 & 1, 4

local min(s): btw -4 & 0, 1 & 4, btw 4 & 5



Sign Change Chart:

Test Point	x^3	$(x-1)$	$(x-5)$	$(x+4)^2$	$(x-4)^2$	$f(x)$
-5	−	−	−	+	+	−
-2	−	−	−	+	+	−
.5	+	−	−	+	+	+
2	+	+	−	+	+	−
4.5	+	+	−	+	+	−
6	+	+	+	+	+	+

Sign Charts & the Test Interval Technique

Name ______________________

Date ________________ **Block** ______

Consider the function $p(x) = (x+4)(x+2)^2(x-2)(x-4)^2$. Note that $p(x)$ is already in *factored form*. The zeros of a polynomial in factored form can be read off without trouble. We have $x = -4, -2, 2$ and 4. The *multiplicities* of -2 and 4 are two. Thus we have four branch points as shown on the chart below.

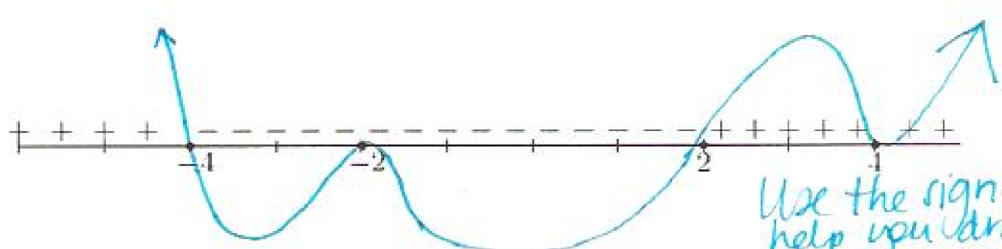
double root

double root

Note that the four branch points divide the number line into five test intervals, $(-\infty, -4), (-4, -2), (-2, 2), (2, 4), (4, \infty)$. Select a *test point* from each interval. Let's take $-5, -3, 0, 3,$ and 5.

To determine the sign of the function at each test point, build a matrix with test points listed down the side and factors listed along the top. In the current case

test point	$(x+4)$	$(x+2)^2$	$(x-2)$	$(x-4)^2$	$p(x)$
-5	$-$	$+$	$-$	$+$	$+$
-3	$+$	$+$	$-$	$+$	$-$
0	$+$	$+$	$-$	$+$	$-$
3	$+$	$+$	$+$	$+$	$+$
5	$+$	$+$	$+$	$+$	$+$



Use the signs to help you draw the curves.