8-6 The Law of Cosines

Name Key

Date ______________ Block ______

When solving a non-right triangle (finding all missing sides and angles), we need another approach besides the Law of Sines if there is not a side and opposite angle measure given. This approach is called the Law of Cosines.

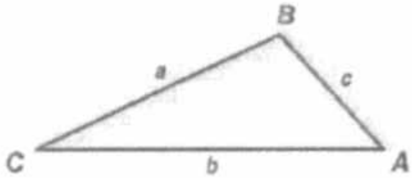
Theorem 8.11 Law of Cosines

If $\triangle ABC$ has lengths a, b, and c, representing the lengths of the sides opposite the angles with measures A, B, and C, then

$a^2 = b^2 + c^2 - 2bc \cos A$,

$b^2 = a^2 + c^2 - 2ac \cos B$, and

$c^2 = a^2 + b^2 - 2ab \cos C$.

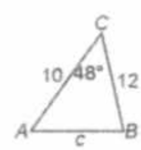


The Law of Cosines can be used when you know either one of the following:

- the measures of 2 sides and their included angle (SAS case)
- the measures of 3 sides (SSS case)

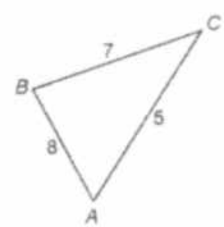
Example 1 **Find c. Round to the nearest tenth.**

$c^2 = a^2 + b^2 - 2ab \cos C$	Law of Cosines
$c^2 = 12^2 + 10^2 - 2(12)(10)\cos 48°$	$a = 12$, $b = 10$, $m\angle C = 48$
$c = \sqrt{12^2 + 10^2 - 2(12)(10)\cos 48°}$	Take the square root of each side.
$c \approx 9.1$	Use a calculator.



Example 2 **Find $m\angle A$. Round to the nearest degree.**

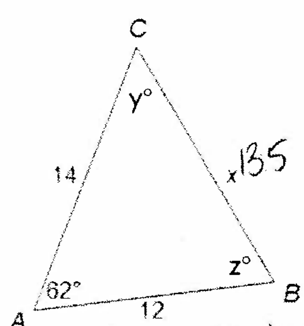
$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$7^2 = 5^2 + 8^2 - 2(5)(8)\cos A$	$a = 7$, $b = 5$, $c = 8$
$49 = 25 + 64 - 80 \cos A$	Multiply.
$-40 = -80 \cos A$	Subtract 89 from each side.
$\frac{1}{2} = \cos A$	Divide each side by −80.
$\cos^{-1} \frac{1}{2} = A$	Use the inverse cosine.
$60° = A$	Use a calculator.



Solve each triangle below by finding the values of x, y, and z. Round sides and angles to the nearest tenth.

1\.

$x =$ 13.5 $y =$ 51.7° $z =$ 66.3°



$x^2 = 14^2 + 12^2 - 2(14)(12)\cos 62$

$x = 13.5$

$14^2 = 12^2 + 13.5^2 - 2(12)(13.T)\cos z$

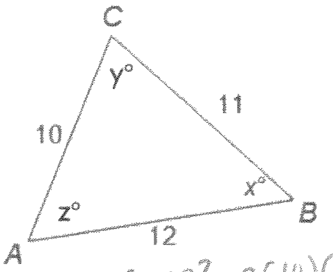
$196 = 326\ 25 - 324 \cos z$

$\frac{-130.25}{-324} = \cos z$

$z = 66.3°$

180
−62
−66.3

2.



x = 51.3° y = 69.5° z = 59.2°

$11^2 = 10^2 + 12^2 - 2(10)(12)\cos z$

$\frac{-123}{-240} = \cos z = 59.2$

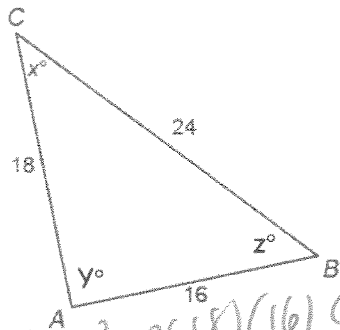
$10^2 = 12^2 + 11^2 - 2(12)(11)(\cos x)$

$\frac{-165}{-264} = 51.3$

$12^2 = 10^2 + 11^2 - 2(10)(11)(\cos y)$

$\frac{-77}{-220} = \cos y = 69.5$

3.



x = 41.8° y = 89.6° z = 48.6°

$24^2 = 18^2 + 16^2 - 2(18)(16)\cos y$

$\frac{-4}{-576} = \cos y = 89.6$

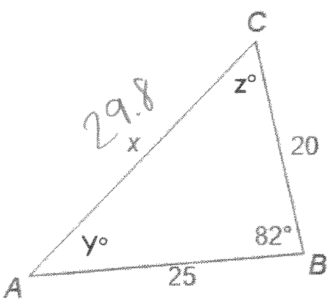
$18^2 = 24^2 + 16^2 - 2(24)(16)\cos z$

$\frac{-508}{-768} = \cos z = 48.6$

$16^2 = 24^2 + 18^2 - 2(24)(18)\cos x$

$\frac{-644}{-864} = \cos x = 41.8$

4.



x = 29.8 y = 41.7° z = 56.2°

$x^2 = 25^2 + 20^2 - 2(25)(20)\cos 82$

$= 29.8$

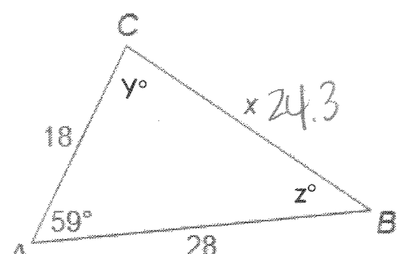
$20^2 = 29.8^2 + 25^2 - 2(29.8)(25)\cos y$

$\frac{-1113.04}{-1490} = \cos y = 41.7$

$25^2 = 29.8^2 + 20^2 - 2(29.8)(20)\cos z$

$\frac{-663.04}{-1192} = \cos z = 56.2$

5.



x = 24.3 y = 81.4° z = 39.5°

$x = \sqrt{18^2 + 28^2 - 2(18)(28)\cos 59}$

$x = 24.3$

$18^2 = 28^2 + 24.3^2 - 2(28)(24.3)\cos z$

$\frac{-1050.49}{-1360.8} = \cos z = 39.5$

$28^2 = 18^2 + 24.3^2 - 2(18)(24.3)\cos y$

$\frac{-130.49}{-874.8} = \cos y = 81.4$