8-6 The Law of Sines

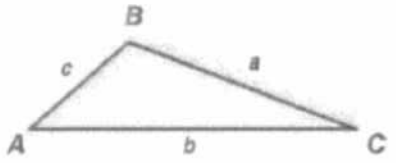
Name Master Eq
Date _____ Block _____

When solving a triangle (finding all missing sides and angles), we need another approach if the triangle is not a right triangle. One method is called the Law of Sines.

Theorem 8.10 Law of Sines

If $\triangle ABC$ has lengths a, b, and c, representing the lengths of the sides opposite the angles with measures A, B, and C, then

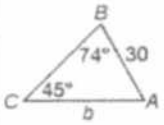
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



The Law of Sines can be used when you know either one of the following:

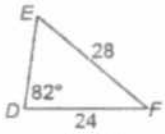
- the measures of 2 angles and the lengths of one of the sides (AAS or ASA cases)
- the measures of 2 sides and the angle opposite one of them (SSA case)

Example 1 Find b. Round to the nearest tenth.



$\frac{\sin C}{c} = \frac{\sin B}{b}$	Law of Sines
$\frac{\sin 45°}{30} = \frac{\sin 74°}{b}$	$m\angle C = 45$, $c = 30$, $m\angle B = 74$
$b \sin 45° = 30 \sin 74°$	Cross Products Property
$b = \frac{30 \sin 74°}{\sin 45°}$	Divide each side by sin 45°.
$b \approx 40.8$	Use a calculator.

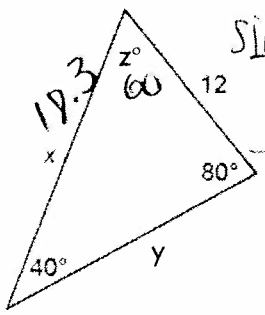
Example 2 Find $m\angle E$. Round to the nearest degree.



$\frac{\sin E}{d} = \frac{\sin E}{e}$	Law of Sines
$\frac{\sin 82°}{28} = \frac{\sin E}{24}$	$d = 28$, $m\angle D = 82$, $e = 24$
$24 \sin 82° = 28 \sin E$	Cross Products Property
$\sin E = \frac{24 \sin 82°}{28}$	Divide each side by 28.
$E = \sin^{-1} \frac{24 \sin 82°}{28}$	Use the inverse sine.
$E \approx 24°$	Use a calculator.

Solve each triangle below by finding the values of x, y, and z.

1.



$\frac{\sin 80}{x} = \frac{\sin 40}{12} = \frac{\sin 60}{y}$

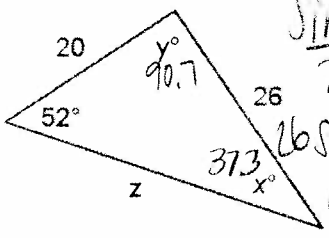
$x \sin 40 = 12 \sin 80$

$x = \frac{12 \sin 80}{\sin 40}$ 18.38

$y \sin 40 = 12 \sin 60$

$y = \frac{12 \sin 60}{\sin 40}$ 16.16

2.



$\frac{\sin 52}{26} = \frac{\sin x}{20} = \frac{\sin 9}{z}$

$26 \sin x = 20 \sin 52$

$\sin x = \frac{20 \sin 52}{26}$ 37

$z \sin 52 = 26 \sin 90$

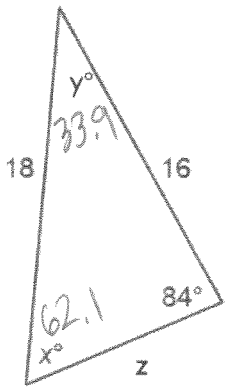
$z = \frac{26 \sin 90.7}{\sin 52}$

32.99

x = 18.3 y = 16.2 z = 60°

x = 37.3° y = 90.7 z = 33.0

3.



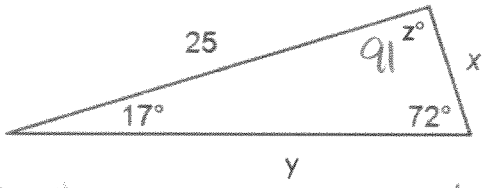
$\frac{\sin x}{16} = \frac{\sin 33.9}{z} = \frac{\sin 84}{18}$

$\sin x = \frac{16 \sin 84}{18}$ $z \sin 84 = 18 \sin 33.9$

$z = \frac{18 \sin 33.9}{\sin 84}$

x = 62.1° y = 33.9 z = 10.1

4.



$\frac{\sin 17}{x} = \frac{\sin 72}{25} = \frac{\sin 91}{y}$

$x \sin 72 = 25 \sin 17$

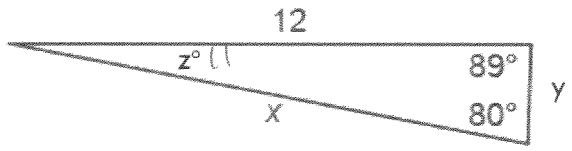
$x = \frac{25 \sin 17}{\sin 72} = 7.68$

$y \sin 72 = 25 \sin 91$

$y = \frac{25 \sin 91}{\sin 72}$ 26.28

x = 7.7 y = 26.3 z = 91°

5.



$\frac{\sin 89}{x} = \frac{\sin 80}{12} = \frac{\sin 11}{y}$

$x \sin 80 = 12 \sin 89$

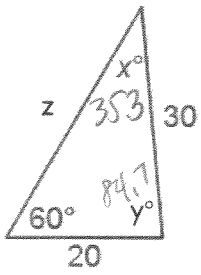
$x = \frac{12 \sin 89}{\sin 80}$ 12.18

$y \sin 80 = 12 \sin 11$

$y = \frac{12 \sin 11}{\sin 80}$ 2.32

x = 12.2 y = 2.3 z = 11°

6.



$\frac{\sin 60}{30} = \frac{\sin x}{20} = \frac{\sin 84.7}{z}$

$\sin x = \frac{20 \sin 60}{30}$ 35.26

$z \sin 60 = 30 \sin 84.7$

$z = \frac{30 \sin 84.7}{\sin 60}$ 34.49

x = 35.3° y = 84.7° z = 34.5°