Permutations & Combinations

Objective: The student will be able to use the Fundamental Counting Principle, permutations, and combinations to determine the number of possible arrangements or selections of items.
SOL AII.12: The student will compute and distinguish between permutations and combinations and use technology for applications.

1. **Fundamental Counting Principle**: helps to find the number of possible outcomes when two or more events occur.
   - **Two events:** If one event can occur in m ways and another event can occur in n ways, then the number of ways that both events can occur is $m \cdot n$.
   - **Three or more events:** If three events can occur in m, n, and p ways, then the number of ways that all three events can occur is $m \cdot n \cdot p$.
2. **Tree Diagram**: uses branches to show all the possible arrangements of objects in a set, which represents the number of ways to perform a task.

A. An ice cream shop offers sundaes consisting of two different types of ice cream, vanilla or chocolate, and two different types of syrup, strawberry or chocolate. How many different types of sundaes are available at the ice cream shop?

Ice Cream Flavors — **Syrups**

Chocolate → Strawberry
Chocolate → Chocolate
Vanilla → Strawberry
Vanilla → Chocolate

There are 4 possible choices:

Chocolate ice cream and strawberry syrup
Chocolate ice cream and chocolate syrup
Vanilla ice cream and strawberry syrup
Vanilla ice cream and chocolate syrup

B. In the problem above, suppose there are 5 possible choices for ice cream and 4 possible choices for syrups. How many different types of sundaes would be available at the ice cream shop?

$5 \cdot 4 = 20$

C. A pet shop sells fish tanks in three different sizes (small, medium, and large), each of which are available in two different shaped bases (pentagon and rectangle). How many different fish tanks are available?

$3 \cdot 2 = 6$

D. Suppose a meal consists of an appetizer, an entrée, and a dessert. Find the total number of different meals from which you can choose if there are five appetizers, three entrees, and six desserts.

$5 \cdot 3 \cdot 6 = 90$

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3. **Permutation:** a group of n objects or people arranged in a certain order where the **order** of the objects **DOES MATTER.**
   ....This is when the same two objects arranged in a different order is considered two different choices.

   - If the combination to a safe is 4-7-2, does the order matter? **YES!**
     In order for a combination to work, it has to be exactly 4-7-2; 7-2-4 or 2-4-7 would NOT work!
   - You can use the fundamental counting principle to find the number of permutations of A, B, and C. There are 3 choices for the first letter. After the first letter has been chosen, 2 choices remain for the second letter. Finally after the first two letters have been chosen, there is only 1 choice remaining for the final letter, so the number of permutations is 3 • 2• 1 = 6. This expression 3 • 2• 1 can be written as 3!
   - Therefore, there are 6 permutations of the letters A, B, and C: ABC, ACB, BAC, BCA, CAB, & CBA
   - The symbol ! is the factorial symbol. **$n! = n(n-1)(n-2)(n-3)$ .... until it is (1)**

CALCULATOR METHOD: n!: Type n → MATH → PRB → 4 → ENTER

A. $5!$ $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

B. $\frac{10!}{8!}$ $\frac{10 \cdot 9 \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$ 90

C. $\frac{9!}{(9-5)!}$ $\frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$ 15,120

- The number of permutations of n distinct objects is n!
- The number of permutations of n different objects taken r at a time is denoted by: $_nP_r = \frac{n!}{(n-r)!}$

CALCULATOR METHOD: $_nP_r$: Type n → MATH → PRB → 2 → r → ENTER

A. $_{14}P_{10}$ $\frac{14!}{4!}$ 3,632,428,800

B. $_6P_4$ $\frac{6!}{2!} = 360$

C. Maggie has eight different colored beads. She will pick six beads to thread with string for a necklace. How many different necklaces can she make in which the order of the beads matters?

$_8P_6 = 20{,}160$

D. How many five digit password numbers can be created using the digits 1, 3, 4, 6, 7, and 9? Assume you can use each digit only once.

$_6P_5 = 720$

E. DJ Boom Boom will create a new CD using eight original songs. In how many different ways can she arrange the songs on the CD?

$_8P_8 = 40{,}320$

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4. **Combination:** a selection of r objects taken from a group of n distinct objects where the order **DOES NOT MATTER.**

This is when the same two objects arranged in a different order is NOT considered two different choices.

- If a fruit salad is made from a combination of apples, grapes and bananas, does the order of the fruit matter? **NO!** We don't care what order the fruits are put in the bowl, when you mix them up, it is the same fruit salad!
- The number of combinations of r objects taken from a group of n distinct objects is denoted by: $_nC_r = \frac{n!}{r!(n-r)!}$

CALCULATOR METHOD: $_nC_r$: Type n → MATH → PRB → 3 → r → ENTER

A. $_{14}C_{10}$ $\frac{14!}{10!\,4!}$ 1001 $\frac{14 \cdot 13 \cdot 12 \cdot 11}{4 \cdot 3 \cdot 2 \cdot 1}$

B. $_6C_4$ $\frac{6!}{4!\,2!}$ $\frac{6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 15$

C. A band director must choose five drummers, out of nine, to march in a parade. In how many different ways can the director line up the five drummers choosing from nine drummers?

$_9C_5 = 126$

D. The Mr. Smoothie's shope has five different types of fresh fruit available. A Supreme smoothie is a blend of three different fruits. How many different Supreme smoothies are possible?

$_5C_3$ 10

E. Coach Hernandez can select seven of the ten girls trying out for the varsity soccer team. In how many ways can he pick his new team members?

$_{10}C_7 = 120$

Combinations & Permutations: What's the Difference?

If the order **doesn't** matter, it's a **Combination**

If the order **does** matter, it is a **Permutation**

A Permutation is an ordered Combination