

Geometry SOL Review: Reasoning, Lines, and Transformations

Master E
Adapted from NCTM's "Reasoning Skills Practice 2014"

SOL G.1: The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include a) identifying the converse, inverse, and contrapositive of a conditional statement; b) translating a short verbal argument into symbolic form; c) using Venn diagrams to represent set relationships; and d) using deductive reasoning, including the law of syllogism.

Hints and Notes

Conditional:	Given Statement:	If p, then q	$p \rightarrow q$
Inverse:	Negate the Statement:	If not p, then not q	$\sim p \rightarrow \sim q$
Converse:	Switch the Hypothesis & Conclusion:	If q, then p.	$q \rightarrow p$
Contrapositive:	Negate the Converse:	If not q, then not p.	$\sim q \rightarrow \sim p$
Biconditional:	Conditional & Converse both true:	p if and only if q.	$p \leftrightarrow q$

Symbols:

\sim	not
\wedge	and
\vee	or
\rightarrow	if then, implies
\leftrightarrow	if and only if
\therefore	therefore

Law of Syllogism: If $a \rightarrow b$ and $b \rightarrow c$, then $a \rightarrow c$.

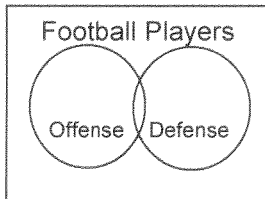
Law of Detachment: If $a \rightarrow b$ is true. Given a is true, then conclude that b is true.

logically equivalent
 INV
 CONV
 COND
 CONTRA

PRACTICE G.1

- Which is the converse of the sentence, "If Sam leaves, then I will stay."?
 A If I stay, then Sam will leave.
 B If Sam does not leave, then I will not stay
 C If Sam leaves then I will not stay.
 D If I do not stay, then Sam will not leave

- According to the Venn Diagram, which is true?
 A Some football players play offense and defense
 B All football players play defense
 C No football players play offense and defense
 D All football players play offense or defense



- Let a represent "x is an even number."
 Let b represent "x is a multiple of 4."
 When $x = 10$, which of the following is true?

- A $a \wedge b$
- B $a \wedge \sim b$
- C $\sim a \wedge b$
- D $\sim a \wedge \sim b$

- Which statement is the inverse of "If the waves are small, I do not go surfing"?

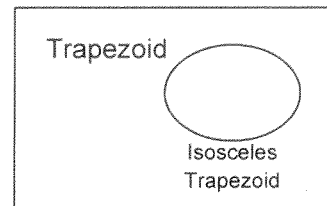
- A If the waves are not small, I do not go surfing.
- B If I do not go surfing, the waves are small.
- C If I go surfing, the waves are not small.
- D If the waves are not small, I go surfing.

- "If the baseball strike does not end, the World Series will not be played" What conclusion can be drawn from these statements?

"If negotiations fail, the baseball strike will not end."

- A If the baseball strike ends, the World Series will not be played
- B If negotiations do not fail, the baseball strike will not end.
- C If negotiations fail, the World Series will not be played
- D If negotiations fail, the World Series will be played

- According to the Venn diagram, which statement is true?



- A No trapezoids are isosceles trapezoids
- B Some trapezoids are isosceles trapezoids
- C All trapezoids are isosceles trapezoids
- D Some isosceles trapezoids are parallelograms

- Which statement is logically equivalent to the true statement

"If it is warm, then I will go swimming."

p *q*

- A If I go swimming, then it is warm.
- B If it is warm, then I do not go swimming.
- C If I do not go swimming, then it is not warm.
- D If it is not warm, then I do not go swimming.

$\sim p \rightarrow \sim q$

SOL G.2: The student will use the relationships between angles formed by two lines cut by a transversal to a) determine whether two lines are parallel; b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

HINTS AND NOTES

Parallel lines – have the same identical slope
 Perpendicular lines – have negative reciprocal slopes

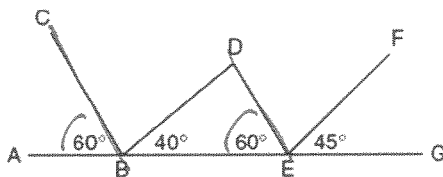
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

To prove that 2 lines are parallel, you must be able to prove one of the following:

- Corresponding, alternate interior, or alternate exterior angles are congruent
- consecutive interior or consecutive exterior angles are supplementary
- both lines are perpendicular to the same line
- the two lines have the same slope

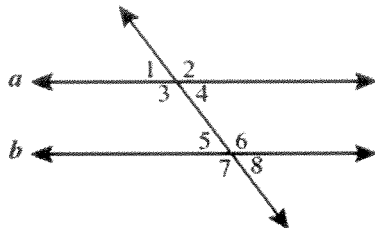
PRACTICE G.2

1. Using the information in the diagram, which is true?



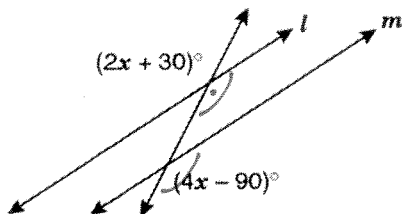
- A $\overline{BD} \parallel \overline{EF}$
- B $\overline{BD} \parallel \overline{DE}$
- C $\overline{CB} \parallel \overline{BD}$
- D $\overline{CB} \parallel \overline{DE}$**

2. Line a is parallel to line b if



- ~~A $\angle 1 \cong \angle 4$~~
- ~~B $\angle 1 \cong \angle 7$~~
- ~~C $\angle 1 \cong \angle 6$~~
- D $\angle 1 \cong \angle 5$**

3. What value for x will show that lines l and m are parallel?



- A 60**
- B 40
- C 30
- D 25

$$4x - 90 = 2x + 30$$

$$2x = 120$$

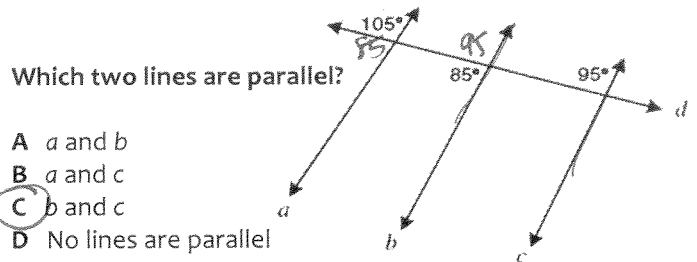
$$x = 60$$

4. Prove that line l is parallel to line m using slope.

Slope of Line l: $-\frac{3}{9}$
 Slope of Line m: $-\frac{1}{3}$

Why are they parallel?
 b/c their slopes are the same

5. In this diagram, line d cuts three lines to form the angles shown.



Which two lines are parallel?

- A a and b
- B a and c
- C b and c**
- D No lines are parallel

6. Which statement describes the lines whose equations are $y = \frac{1}{3}x + 12$ and $6y = 2x + 6$?

$$y = \frac{2}{6}x + 1 = \frac{1}{3}x + 1$$

- A They are segments
- B They are perpendicular to each other
- C They intersect
- D They are parallel to each other**

TEI Hot Spot Item – You will be asked to plot points on a coordinate plane. If you don't use the "arrow" key to plot the points, your answer will not be considered answered. "AAA" (Always use the arrow key). DO NOT USE THE "DOT" KEY. Make sure you plot all points or the problem will be considered incorrect.

SOL G.3: The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include a) investigating and using formulas for finding distance, midpoint, and slope; b) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and c) determining whether a figure has been translated, reflected, or rotated.

HINTS AND NOTES

Midpoint: $(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2})$

Slope: $\frac{y_2 - y_1}{x_2 - x_1}$

Distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Reflection – flips over an axis of symmetry

Dilation – shrinks or blows up

Translation – glides by adding or subtracting to both the x and y coordinate

Rotation – turns around a point

Line of symmetry: a line that can cut an object so that if it is folded all sides and angles will match perfectly.

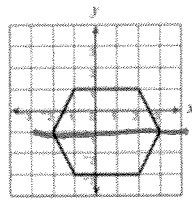
PRACTICE G.3

1. What is the slope of the line through (-2, 3) and (1,1)?

- A $-\frac{3}{2}$
 - B $-\frac{2}{3}$
 - C $\frac{1}{2}$
 - D 2
- $\frac{1-3}{1-(-2)} = \frac{-2}{3}$

2. The hexagon in the drawing has a line of symmetry through –

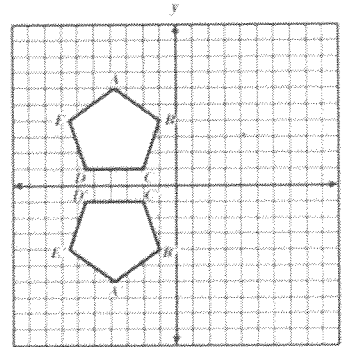
- A (-1, -3) and (2, 1)
- B (1, 1) and (1, -3)
- C (2, 3) and (2, -3)
- D (-2, -1) and (3, -1)



3. Consider the figure provided. Which of the following is a rotation in the plane of the given figure?

- A
- B
- C
- D

4. The polygon A'B'C'D'E' is - ?



- A a translation of ABCDE across the x-axis
- B a 180° clockwise rotation of ABCDE about the origin
- C a reflection of ABCDE across the y-axis
- D a reflection of ABCDE across the x-axis

5. Which point is the greatest distance from the origin?

- A (0, -2) 2
- B (0, -5) 5
- C (-2, 0) 2
- D (0, -12) 12



6. Which polygon shown below has only one line of symmetry?

- A
- C
- B
- D

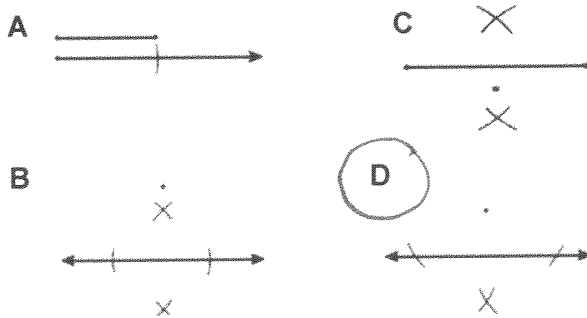
SOL G.4: The student will construct and justify the constructions of a) a line segment congruent to a given line segment; b) the perpendicular bisector of a line segment; c) a perpendicular to a given line from a point not on the line; d) a perpendicular to a given line at a point on the line; e) the bisector of a given angle; f) an angle congruent to a given angle; and g) a line parallel to a given line through a point not on the given line.

HINTS and NOTES

Look at the examples of Constructions on your Geometry Study Guide!

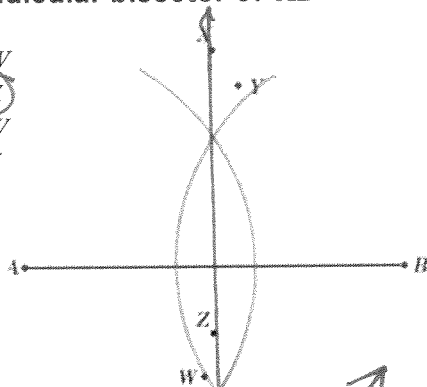
PRACTICE G.4

1. Which drawing shows the arcs for a construction of a perpendicular segment to a line from a point not on the line?

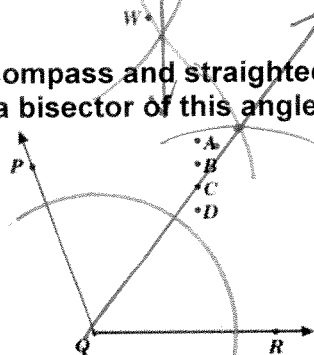


2. Which pair of points determines the perpendicular bisector of \overline{AB}

- A X, W
- B X, Z**
- C Y, W
- D Y, Z



3. Use your compass and straightedge to construct a bisector of this angle.



Which point lies on the bisector?

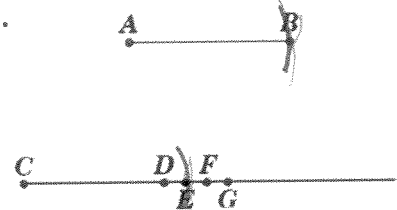
- A A
- B B
- C C**
- D D

4. The drawing shows the arcs used to construct –



- A a bisector of a given angle
- B an angle congruent to a given angle**
- C a bisector of a given line
- D a perpendicular of a line at a point on the line

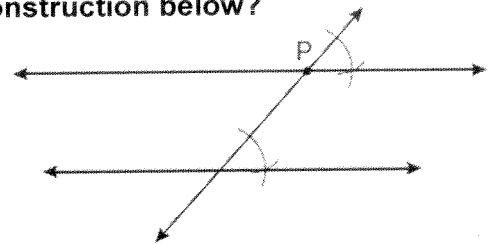
5. Use your compass to answer the following question.



Which line segment is congruent to \overline{AB} ?

- A \overline{CD}
- B \overline{CE}**
- C \overline{CF}
- D \overline{CG}

6. Which geometric principle is used to justify the construction below?



- A A line perpendicular to one of two parallel lines is perpendicular to the other
- B Two lines are perpendicular if they intersect to form congruent adjacent angles
- C When two lines are intersected by a transversal and alternate interior angles are congruent, the lines are parallel
- D When two lines are intersected by a transversal and the corresponding angles are congruent, the lines are parallel**