

SOL G.9: The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.

**HINTS and NOTES**

**Quadrilateral** – polygon with four sides, whose interior angle sum is  $360^\circ$

**Parallelogram:**

- Opposite sides  $\cong$
- Opposite sides parallel
- Opposite angles  $\cong$
- Consecutive angles are supplementary
- Diagonals bisect each other

**Rectangle:**

- Opposite sides  $\cong$
- Opposite sides parallel
- Opposite angles  $\cong$
- Consecutive angles are supplementary
- Diagonals bisect each other
- Four right angles ( $90^\circ$ )
- Diagonals are  $\cong$

**Rhombus:**

- Opposite sides  $\cong$
- Opposite sides parallel
- Opposite angles  $\cong$
- Consecutive angles are supplementary
- Diagonals bisect each other
- Four  $\cong$  sides
- Diagonals are perpendicular

**Square:**

- Opposite sides  $\cong$
- Opposite sides parallel
- Opposite angles  $\cong$
- Consecutive angles are supplementary
- Diagonals bisect each other
- Four right angles
- Four  $\cong$  sides
- Diagonals are  $\cong$
- Diagonals are perpendicular

**Kite:**

- Diagonals are perpendicular
- One pair of opp. angles are  $\cong$

**Trapezoid:**

- One pair of opposite sides are parallel
- Two pairs of consecutive interior angles are supplementary

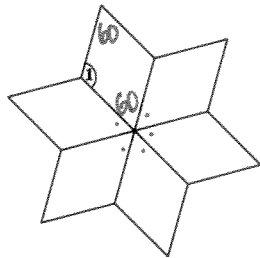
**Isosceles trapezoid:**

- One pair of opposite sides are parallel
- Two pairs of consecutive interior angles are supplementary
- One pair of opposite sides parallel
- Two pairs of consecutive angles are congruent
- Diagonals bisect each other

**PRACTICE G.9**

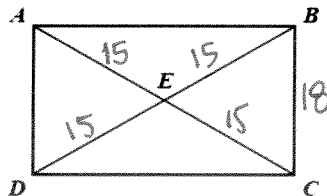
1. The design for a quilt piece is made up of 6 congruent parallelograms. What is the measure of  $\angle 1$  ?

- A  $150^\circ$
- B  $120^\circ$**
- C  $60^\circ$
- D  $30^\circ$



2. Figure ABCD is a rectangle.  $\overline{AC}$  and  $\overline{BD}$  are diagonals.  $AC = 30$  meters and  $BC = 18$  meters. What is the length of  $\overline{DE}$  ?

- A 8 meters
- B 10 meters
- C 15 meters**
- D 24 meters

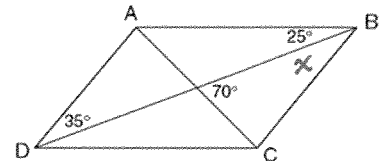


3. Which quadrilateral could have diagonals that are congruent but do not bisect each other?

- ~~A~~ rectangle - has  $\cong$  diags, but they bisect each other!
  - ~~B~~ rhombus - diags. bi. each other, but aren't  $\cong$
  - C** trapezoid - diags. don't bi. each other, but are  $\cong$
  - ~~D~~ parallelogram - diags. bi. each other, but aren't  $\cong$
- alt. isos trap. would have  $\cong$  diags.
- diags  $\neq$  but do bisect each other
- 

4. In parallelogram ABCD, what is  $m\angle DBC$  ?

- A  $25^\circ$
- B  $35^\circ$**
- C  $45^\circ$
- D  $70^\circ$

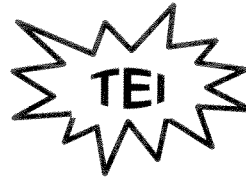
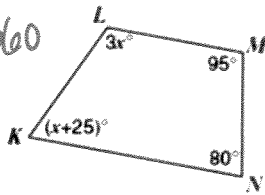


alt. int.  $\neq$  S

5. Given quadrilateral KLMN, what is the value of  $x$ ?

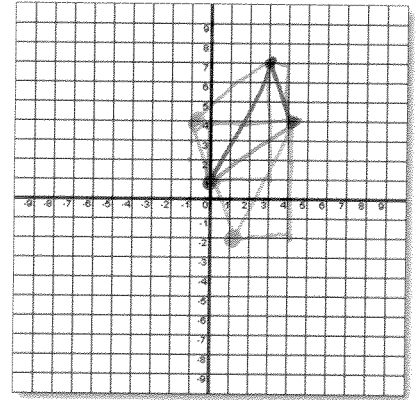
- A 35
- B 40**
- C 45
- D 50

$4x + 200 = 360$   
 $4x = 160$   
 $x = 40$



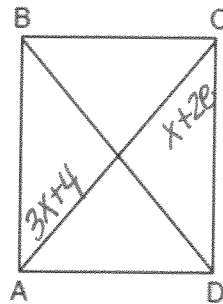
**Hot Spot Item** – You will be asked to plot points on a coordinate plane. If you don't use the "arrow" key to plot the points, your answer will not be considered answered. "AAA" (Always use the arrow key) DO NOT USE THE "DOT" KEY. Make sure you plot all points or the problem will be considered incorrect.

6. Three vertices of a parallelogram have coordinates  $(0, 1)$ ,  $(3, 7)$ , and  $(4, 4)$ . Place a point on the graph that could represent the fourth vertex of the parallelogram.  $(1, -2)$  or  $(-1, 4)$



Put your answer in the box. These are open-ended questions. Work them out write your answer in the box (on the computer you would type your answer in the box being sure to put it in appropriate form, simplest fraction, decimal, etc.) For our purposes, you will write your answer in the box.

7. In the accompanying diagram of rectangle ABCD,  $m\angle BAC = 3x + 4$  and  $m\angle ACD = x + 28$ . What is  $m\angle CAD$ ?

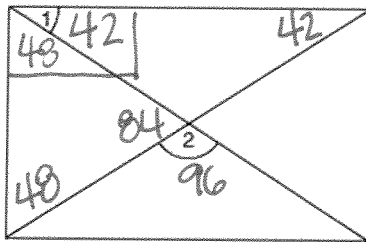


$3x + 4 = x + 28$   
 $2x = 24$   
 $x = 12$



50

8. As shown in the accompanying diagram a rectangular gate has two diagonal supports. If  $m\angle 1 = 42$ , what is  $m\angle 2$ ?

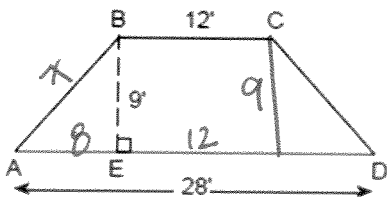


$180$   
 $-84$   


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96

9. The cross section of an attic is in the shape of an isosceles trapezoid, as shown. If the height of the attic is 9 feet,  $BC = 12$  feet, and  $AD = 28$  feet, find the length of  $AB$  to the nearest foot.



$8^2 + 9^2 = x^2$   
 $145 = x^2$   
 $12.04 = x$

12

SOL G.10: The student will solve real-world problems involving angles of polygons.

HINTS and NOTES

Polygon	# of sides	Sum of the interior angles
Triangle	3	$180^\circ$
Quadrilateral	4	$360^\circ$
Pentagon	5	$540^\circ$
Hexagon	6	$720^\circ$
Heptagon	7	$900^\circ$
Octagon	8	$1080^\circ$
Nonagon	9	$1260^\circ$
Decagon	10	$1440^\circ$

Sum of the exterior angles of any polygon is always =  $360^\circ$ .

Remember: Number of sides = number of interior angles

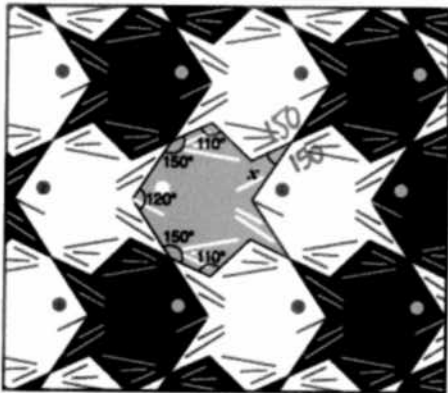
Equilateral – means all sides are congruent.

Equiangular - means all angles are congruent.

Regular – means all sides and all angles are equal in measure (congruent).

PRACTICE G.10

1. Some of the angle measures are given for one of the fish shaped polygons in this tessellation. What is the value of  $x$ ?



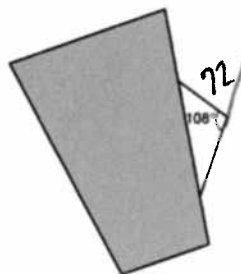
- A  $30^\circ$
- B  $40^\circ$
- C  $45^\circ$
- D  $60^\circ$

$$300 + 2x = 360$$

$$2x = 60$$

$$x = 30$$

2. In the drawing, a regular polygon is partially covered by the trapezoid. How many sides does the covered polygon have?

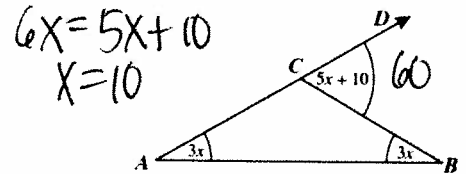


- A 8
- B 6
- C 5
- D 4

$$\frac{360}{72} = 5$$

3. The figure has angle measures as shown. What is the measure of  $\angle BCD$ ?

- A  $30^\circ$
- B  $60^\circ$
- C  $80^\circ$
- D  $120^\circ$



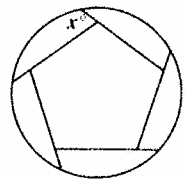
$$6x = 5x + 10$$

$$x = 10$$

4. A floor tile is designed with a regular pentagon in the center of the tile with its sides extended. What is the value of  $x$ ?

- A  $72^\circ$
- B  $90^\circ$
- C  $110^\circ$
- D  $120^\circ$

$$\frac{360}{5}$$

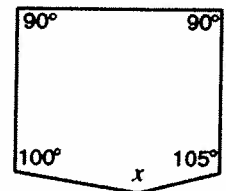


5. What is the value of  $x$  in the pentagon?

- A  $90^\circ$
- B  $155^\circ$
- C  $245^\circ$
- D  $335^\circ$

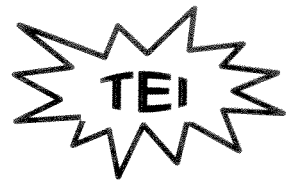
$$(5-2)(180)$$

$$3(180)$$



$$540 = x + 385$$

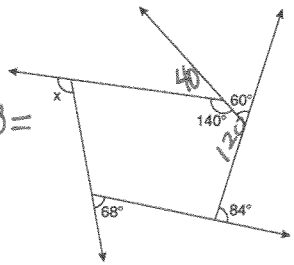
$$155 = x$$



6. The pentagon in the diagram below is formed by five rays. What is the degree measure of  $\angle x$ ?

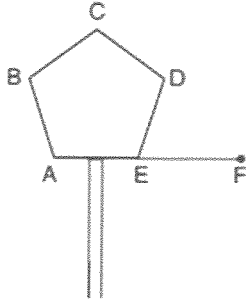
$$x + 40 + 60 + 84 + 68 = 360$$

$$x + 252 = 360$$



108

7. One piece of the birdhouse that Natalie is building is shaped like a regular pentagon, as shown in the accompanying diagram. If side AE is extended to point F, what is the measure of exterior angle DEF?



$$\frac{360}{5}$$

72

8. What is the measure of an interior and exterior angle of a regular octagon?

Interior angle

135

Exterior angle

45

$$\frac{135}{45}$$

**Geometry SOL Review: Polygons, Circles, & 3-D Figures**

Edited from VBCPS "Review Skills Notes Practice 2014"

SOL G.11: The student will use angles, arcs, chords, tangents, and secants to a) investigate, verify, and apply properties of circles; and c) find arc lengths and areas of sectors in circles.

**HINTS and NOTES**

- A circle always measures  $360^\circ$ . A semi-circle always measures  $180^\circ$ . Radius =  $\frac{1}{2}$  diameter
- Radius – a connects the center of a circle to any point on the circle Diameter = twice the radius
- Diameter – a segment that connects any two points on a circle and passes through the center
- A diameter cuts a circle in half (makes two semi-circles) Arc – a piece of a circle
- Chord – a segment that connects two points on a circle (the diameter is the longest chord)
- Tangent – a line, segment, or ray that intersects (touches) a circle at one point (the point of tangency)
- Inscribed – means inside
- Ex: a circle inscribed in a square
- Circumscribed – means outside
- Ex: the square is circumscribed about the circle



**Angle Formulas:**

- Tangent  $\perp$  radius
- Central angle = intercepted arc
- Inscribed angle =  $\frac{1}{2}$  intercepted arc
- Outside angle =  $\frac{1}{2}$  (big arc – little arc)
- Inside angle (makes an x) =  $\frac{1}{2}$  (intercepted arc + intercepted arc)
- Magic hat (formed by two tangents to a circle): top of the head + tip of the hat =  $180^\circ$  and sides are congruent

When two chords intersect in a circle (making an x): **Figure 1**  
part • part = part • part:  $AE \bullet EC = DE \bullet EB$

When two secants intersect outside a circle: **Figure 2**  
Outside part • whole = outside part • whole:  $AB \bullet AC = AE \bullet AD$

When a secant and a tangent intersect outside a circle: **Figure 3**  
Tangent squared = outside part • whole:  $AB^2 = BC \bullet BD$

Figure 1

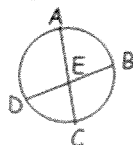


Figure 2

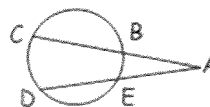


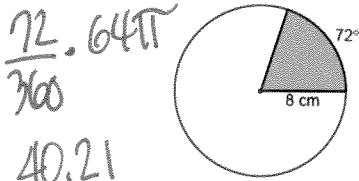
Figure 3



**PRACTICE G.11**

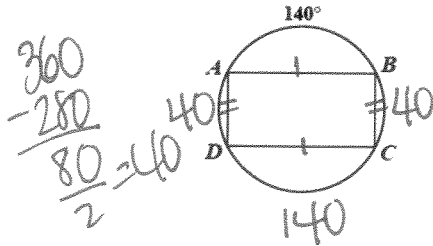
1. A circle has a radius of 8 centimeters. The measure of the arc of the shaded region is  $72^\circ$ . Which is closest to the area of the shaded region of the circle?

- A  $160.8 \text{ cm}^2$   
 B  $50.3 \text{ cm}^2$   
 C  $40.2 \text{ cm}^2$   
 D  $10.1 \text{ cm}^2$



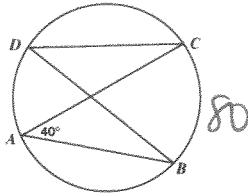
2. Rectangle  $ABCD$  is inscribed in a circle. If the measure of the  $\widehat{AB}$  is  $140^\circ$ , what is the measure of  $\widehat{BC}$ ?

- A  $30^\circ$   
 B  $40^\circ$   
 C  $60^\circ$   
 D  $80^\circ$



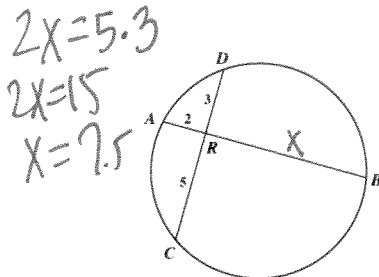
3. If  $m\angle CAB = 40^\circ$ , what is  $m\angle CDB$ ?

- A  $20^\circ$   
 B  $40^\circ$   
 C  $60^\circ$   
 D  $80^\circ$



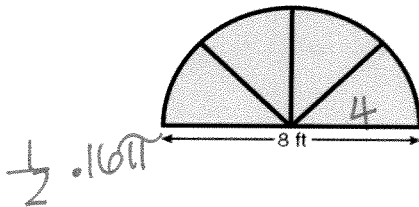
4. Chords  $\overline{AB}$  and  $\overline{CD}$  intersect at  $R$ . Using the values shown in the diagram, what is the measure of  $\widehat{RB}$ ?

- A 6  
 B 7.5  
 C 8  
 D 9.5



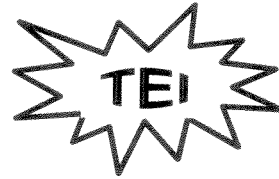
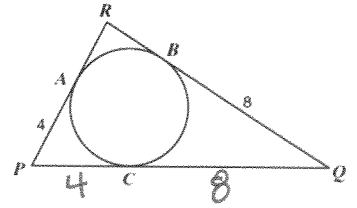
5. This is a sketch of a stained-glass window in the shape of a semicircle. Ignoring the seams, how much glass is needed for the window?

- A  $4\pi \text{ ft}^2$   
 B  $8\pi \text{ ft}^2$   
 C  $12\pi \text{ ft}^2$   
 D  $16\pi \text{ ft}^2$



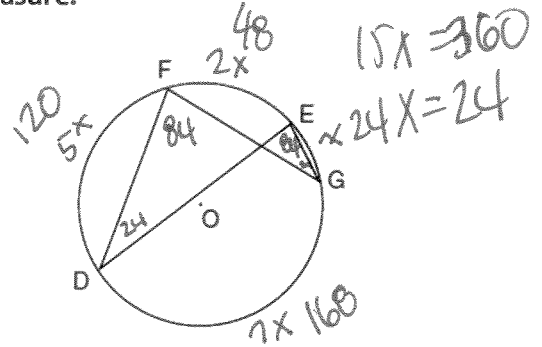
6.  $A, B,$  and  $C$  are points of tangency.  $AP = 4$  and  $BQ = 8$ . What is the measure of  $\widehat{PQ}$ ?

- A 4  
 B 8  
 C 12  
 D  $\sqrt{32}$



Highlight each correct answer. These questions give you choices. You must click on each correct answer and make sure you mark every answer that is correct. If you forget one, it will be incorrect. Circle each correct answer. More than one answer may be selected.

7. In the diagram below of circle  $O$ , chords  $\overline{DF}, \overline{DE}, \overline{FG},$  and  $\overline{EG}$  are drawn such that  $m\widehat{DF} : m\widehat{FE} : m\widehat{EG} : m\widehat{GD} = 5 : 2 : 1 : 7$ . Identify one pair of inscribed angles that are congruent to each other by highlighting the angles and give their measure.



$\angle D$	$\angle F$	$\angle E$	$\angle G$
$m\angle D = 24$	$m\angle F = 84$	$m\angle E = 84$	$m\angle G = 24$

SOL G.12: The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

**HINTS and NOTES**

Circle- locus of points equidistant from a given point (center)

Standard Form Equation of Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Coordinates of the center of the circle:

$$(h, k)$$

The length of the radius (r):

Take the square root of  $r^2$

**PRACTICE G.12**

1. Write the standard equation of a circle with its center at the origin and radius of 7.

A  $x^2 + y^2 = 49$

B  $x^2 + y^2 = 14$

C  $x^2 + y^2 = 7$

D  $\frac{x^2}{14} + \frac{y^2}{14} = 1$

2. Write the standard equation of a circle with center (4, -4) and radius 4.

A  $(x-4)^2 + (y+4)^2 = 16$

B  $(x+4)^2 + (y-4)^2 = 4$

C  $(x+4)^2 - (y-4)^2 = 4$

D  $(x-4)^2 + (y-4)^2 = 16$

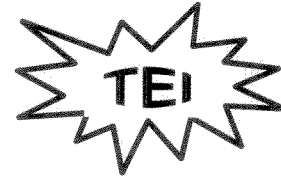
3. The standard equation of a circle with center (-4, 3) and radius 7 is \_\_\_\_.

A  $(x-4)^2 + (y+3)^2 = 7$

B  $(x+4)^2 + (y-3)^2 = 49$

C  $(x-4)^2 + (y-3)^2 = 49$

D  $(x+4)^2 + (y-3)^2 = 7$



4. Identify the center, radius, and diameter of a circle with the following standard equation.

$$(x-4)^2 + (y+10)^2 = 100$$

$(4, -10)$

Center

10

Radius

20

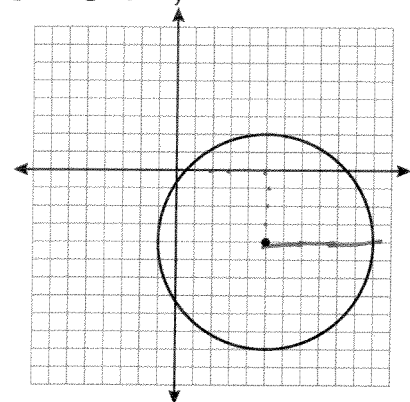
Diameter

5. Write the standard equation of a circle with center (-2, 5) and the point (4, 13) on the circle.

$$(x+2)^2 + (y-5)^2 = 100$$

$$r = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$$

6. Write a standard equation of the circle graphed in the diagram graphed below.



$C(5, -4)$   
 $r=6$

$$(x-5)^2 + (y+4)^2 = 36$$

SOL G.13: The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

**HINTS and NOTES**

\*\*\*Use the formula sheet provided for these problems\*\*\*

**Lateral area** – the amount of area that covers only the sides of a figure (minus the base for a pyramid or cone or two bases if it is a prism or cylinder) - measured in square units

**Surface area** (can also be called total area) – the amount of area that it takes to cover a figure completely - measured in square units

**Volume** (can also be called capacity) – the amount of space you have to fill something - measured in cubic units

**PRACTICE G.13**

1. Rounded to the nearest hundred cubic meters, what is the total capacity (cone and cylinder) of the storage container?

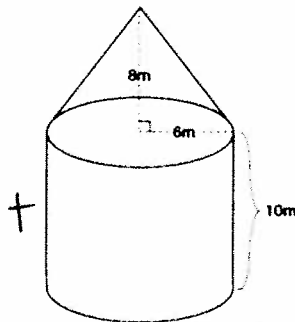
$$\frac{1}{3}\pi(36)(8)$$

A 1,400 cubic meters

B 2,000 cubic meters

C 5,700 cubic meters

D 8,100 cubic meters



$$\pi(36)(10)$$

$$1432.6$$

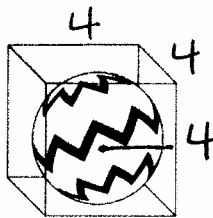
2. A sphere with a 2-inch radius is packed in a cube so that all sides touch. How much empty space is left in the cube?

A 17.8 cubic inches

B 30.5 cubic inches

C 33.5 cubic inches

D 47 cubic inches



$$r=2$$

$$\text{Volume of cube} = 4^3 = 64$$

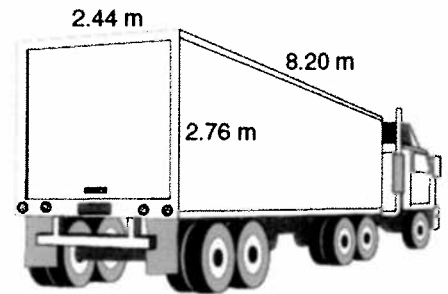
$$\text{Volume of sphere} = \frac{4}{3}\pi(2)^3$$

$$33.5$$

$$64 - 33.5$$

$$\underline{\underline{30.5}}$$

3. The cargo space of the truck is 2.44 meters wide, 2.76 meters high and 8.20 meters long. How many cubic meters of cargo space does the truck have?



A 26.80 cubic meters

B 55.22 cubic meters

C 98.75 cubic meters

D 110.44 cubic meters

$$(2.44)(2.76)(8.20)$$

4. A spherical paintball measures 1.5 centimeters in diameter. Approximately how much paint is in it?

A 14.13 cm<sup>3</sup>

B 9.42 cm<sup>3</sup>

C 7.07 cm<sup>3</sup>

D 1.77 cm<sup>3</sup>

$$r = 1.5 \div 2 = .75$$

$$V = \frac{4}{3}\pi(.75)^3$$

$$1.77$$

5. To the nearest gallon, what is the volume of a cylindrical water heater 1.4 feet in diameter and 4 feet tall? (1 cubic foot = 7.48 gallons)

A 34 gal.

B 46 gal.

C 59 gal.

D 132 gal.



$$\pi(.7)^2(4)$$

$$6.16 \text{ ft}^3$$

$$\times 7.48$$

$$46.06$$

**Geometry SOL Review: Polygons, Circles, & 3-D Figures**

*Edited from NCEPS "Review Skills Notes Practice 2014"*

**SOL G.14:** The student will use similar geometric objects in two-or three dimensions to a) compare ratios between side lengths, perimeters, areas, and volumes; b) determine how changes in one or more dimensions of an object affect area and /or volume of the object; c) determine how changes in area and /or volume of an object affect one or more dimensions of the object; and d) solve real-world problems about similar geometric objects.

**HINTS and NOTES:**

**Dimensional changes:**

	area	volume
height		
length		
width		
radius		
diameter		
perimeter		
circumference		
$a$	$a^2$	$a^3$

**Scale factor:** Ratio of the sides  $\rightarrow \frac{a}{b}$

**Ratio of the perimeters = scale factor**  $\rightarrow \frac{a}{b}$

**Ratio of the areas = (scale factor)<sup>2</sup>**  $\rightarrow \frac{a^2}{b^2}$

**Ratio of the volumes = (scale factor)<sup>3</sup>**  $\rightarrow \frac{a^3}{b^3}$

*\*When finding a missing side of similar figures always set up a proportion keeping corresponding parts in the same order. Then cross multiply to solve.*

**PRACTICE G.14**

1. A cylindrical paint can has the capacity of one gallon. For another size can, the height is doubled. What is the capacity of the larger size?

A 2 gal.  
 B 4 gal.  
 C 8 gal.  
 D 16 gal.

$SF = 1:2$        $V = 1:8$

2. If the edge of a cube is  $2x$ , the volume of the cube is:

A  $8x^3$   
 B  $4x^3$   
 C  $2x^3$   
 D  $4x^2$

$(2x)^3 = 8x^3$

3. If the radius of a circle is tripled, then the area of the circle is multiplied by:

A 27  
 B 9  
 C 3  
 D 6

$SF = 1:3$   
 $AF = 1:9$

4. Two similar cones have heights 5 and 20. What is the ratio of their volumes?

A 1:16  
 B 1:64  
 C 1:4  
 D 4:16

$SF = 5:20 = 1:4$        $VF = 1:64$

5. The distance across a river was estimated by making the measurements shown. Which is a good estimate of the distance  $d$ ?

A 9 ft.  
 B 36 ft.  
 C 39 ft.  
 D 69 ft.

$\frac{9}{d} = \frac{12}{52}$   
 $12d = 468$   
 $d = 39$

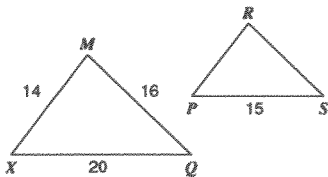


6. The ratio of the circumference of two circles is  $\frac{3}{2}$ . The radius of the smaller circle is 8 inches. What is the radius of the larger circle?

- A  $5\frac{1}{3}$  inches  
 B 6 inches  
 C 9 inches  
 D 12 inches

$CF = 3:2$   
 $\frac{3}{2} = \frac{x}{8}$   
 $2x = 24 \quad x = 12$

7. Which proportion can be used to find the value of  $\overline{PR}$  if  $\triangle XMQ$  is similar to  $\triangle PRS$ ?



- A  $\frac{20}{15} = \frac{14}{PR}$   
 B  $\frac{10}{5} = \frac{7}{PR}$   
 C  $\frac{14}{20} = \frac{15}{PR}$   
 D  $\frac{15}{20} = \frac{14}{PR}$

8. The ratio between the volumes of two spheres is 27 to 8. What is the ratio between their respective radii?

- A 81:64  
 B 27:16  
 C 9:8  
 D 3:2

$VR = 27:8$   
 $CF = \sqrt[3]{27:8} = 3:2$

9. Two triangles are similar. The lengths of the sides of the smaller triangle are 3, 5, and 6, and the length of the longest side of the larger triangle is 18. What is the perimeter of the larger triangle?

- A 14  
 B 18  
 C 24  
 D 42

$\frac{sm}{lg} = \frac{3}{18} = \frac{5}{18} = \frac{6}{18} = Per = 14$   
 $CF = 1:3$   
 $14(3) = 42$

10. A triangle has sides whose lengths are 5, 12, and 13. A similar triangle could have sides with lengths of

- A 6, 8, and 10  
 B 3, 4, and 5  
 C 7, 24, and 25  
 D 10, 24, and 26

11. The ratio of the corresponding sides of similar squares is 1 to 3. What is the ratio of the area of the smaller square to the area of the larger square?

- A  $1:\sqrt{3}$   
 B 1:6  
 C 1:3  
 D 1:9

$(1:3)^2$

12. Describes what happens to the volume of a cone if its radius is doubled while the height is halved. The volume is \_\_\_\_\_.

- A unchanged  
 B doubled  
 C increased by a factor of  $\frac{1}{3}$   
 D not able to be determined



$r = 3$   
 $h = 4$

$\frac{1}{3}\pi(9)(4)$   
 $12\pi$



$r = 6$   
 $h = 2$

$\frac{1}{3}(\pi)(36)(2)$   
 $24\pi$