HW: Solving Quadratic Equations using Square Roots

Name Master 😊 Date________ Block______

MULTIPLYING RADICALS: When multiplying two radicals, the indices must be the same If they are, then multiply the radicands and place the product under one radical sign.

Example 1: $\sqrt{25} \bullet \sqrt{100} = \sqrt{2500} = 50$

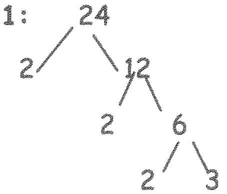
Also notice $5 \bullet 10 = 50$

Example 2: $\sqrt{5} \bullet \sqrt{-5} = \sqrt{-25} = 5i$ ($\sqrt{-1} = i$) It is not real. It is an imaginary number!
Therefore, if one of the radicands is negative and the index is even then there are "no real roots"

SIMPLIFYING RADICALS: A radical is in simplest form when all exponents are positive, no perfect square factor or fraction is left under the radical, and no radicals are in the denominator.
You can simplify a radical 2 different ways:

A. ***Prime Factorization***

Ex. 1:



So, if $24 = 2^3 \bullet 3$, then

$\sqrt{24} = \sqrt{2^3 \bullet 3} =$

$\sqrt{2^2 \bullet 2 \bullet 3} =$

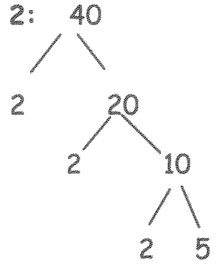
$\sqrt{2^2} \bullet \sqrt{2 \bullet 3} = \boxed{2\sqrt{6}}$

B. ***Perfect Square Factorization***

Since 24 has a factor that's perfect, then break it down as a product of factors and simplify rational factors.

$\sqrt{24} = \sqrt{4} \bullet \sqrt{6} = \boxed{2\sqrt{6}}$

Ex. 2:



So, if $40 = 2^3 \bullet 5$, then

$\sqrt{40a^3b^2} = \sqrt{2^3 \bullet 5 \bullet a^3 \bullet b^2} =$

$\sqrt{2^2 \bullet 2 \bullet 5 \bullet a^2 \bullet a \bullet b^2}$

$2ab\sqrt{2 \bullet 5 \bullet a} = \boxed{2ab\sqrt{10a}}$

Break $40a^3b^2$ in perfect square factors:

$\sqrt{4} \bullet \sqrt{10} \bullet \sqrt{a^2} \bullet \sqrt{a} \bullet \sqrt{b^2} =$

$2 \bullet \sqrt{10} \bullet a \bullet \sqrt{a} \bullet b = \boxed{2ab\sqrt{10a}}$

ADDING OR SUBTRACTING RADICALS: Treat them like you do variables! You can only add or subtract "like" radicals. They must have the same value under the radical sign for them to be alike ($\sqrt{5}$ and $2\sqrt{5}$).

1. $\sqrt{50x^2}$ $\sqrt{25 \cdot 2 \cdot x^2}$

   $5x\sqrt{2}$

2. $\sqrt{48}$ $\sqrt{16 \cdot 3}$

   $4\sqrt{3}$

3. $\sqrt{147x^3y^2}$ $\sqrt{49 \cdot 3 \cdot x^2 \cdot x \cdot y^2}$

   $7xy\sqrt{3x}$

4. $3\sqrt{2} \bullet 4\sqrt{10}$

   $12\sqrt{20} = 12\sqrt{4 \cdot 5}$

   $24\sqrt{5}$

5. $4 + 3\sqrt{5} - 7 + \sqrt{45}$ $\sqrt{9 \cdot 5}$

   $-3 + 3\sqrt{5} + 3\sqrt{5}$

   $-3 + 6\sqrt{5}$

6. $\sqrt{8} \bullet \sqrt{18} \bullet 5\sqrt{4}$

   $5\sqrt{4 \cdot 2 \cdot 2 \cdot 9 \cdot 4}$

   $5 \cdot 2 \cdot 2 \cdot 3 \cdot 2$

   120

DIVIDING RADICALS: $\dfrac{\sqrt{100}}{\sqrt{25}} = \dfrac{10}{5} = 2$ and $\sqrt{\dfrac{100}{25}} = \sqrt{4} = 2$

7. $\sqrt{\dfrac{49}{36}}$ $\dfrac{7}{6}$

8. $\sqrt{\dfrac{225}{289}}$ $\dfrac{15}{17}$

9. $\sqrt{\dfrac{7}{3}} \bullet \sqrt{\dfrac{14}{3}}$ $\dfrac{\sqrt{7 \cdot 7 \cdot 2}}{3} = \dfrac{7\sqrt{2}}{3}$

RATIONALIZING THE DENOMINATOR:

What is $\sqrt{7} \bullet \sqrt{7}$ = ? $\sqrt{7^2}$ = 7
OR $\sqrt{49}$ = 7

So, $\sqrt{218{,}000} \bullet \sqrt{218{,}000}$ = $218{,}000$

We don't usually leave radicals in the denominator of fraction. So, to get a radical out of the denominator, we multiply the fraction as follows:

Example: $\frac{\sqrt{7}}{\sqrt{2}} = \frac{\sqrt{7}}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14}}{\sqrt{2^2}} = \boxed{\frac{\sqrt{14}}{2}}$

10. $\frac{\sqrt{48}}{\sqrt{3}} = \sqrt{16} = 4$

11. $\sqrt{\frac{13}{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{26}}{4}$

12. $\frac{2\sqrt{6}}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{2\sqrt{60}}{10}$

$\frac{2\sqrt{4 \cdot 15}}{10} = \frac{4\sqrt{15}}{10} = \frac{2\sqrt{15}}{5}$

13. $\frac{12\sqrt{18}}{4\sqrt{6}}$ $3\sqrt{3}$

SOLVING QUADRATIC EQUATIONS USING SQUARE ROOTS:

Examples: Solve $5x^2 - 180 = 0$.

$5x^2 = 180$
$x^2 = 36$
$\sqrt{x^2} = \sqrt{36}$
$x = \pm\ 6$

Solve $2(x + 3)^2 = 8$

$(x + 3)^2 = 4$
$\sqrt{(x+3)^2} = \sqrt{4}$
$x+3 = \pm\ 2$
$x = -3 \pm 2$ $\quad -3+2$
$x = -1$ or $x = -5$ $\quad -3-2$

Solve each equation using square roots.

14. $3x^2 - 100 = 332$

$3x^2 = 432$
$x^2 = 144$
$x = \pm 12$

15. $\frac{2}{3}x^2 - 8 = 16$

$\frac{2}{3}x^2 = 24$
$x^2 = 36$
$x = \pm 6$

16. $\frac{1}{2}x^2 - 5 = 5$

$\frac{1}{2}x^2 = 10$
$x^2 = 20$
$x = \pm\sqrt{20} = \pm 2\sqrt{5}$

17. $x^2 + 1 = 3x^2 - 13$

$14 = 2x^2$
$7 = x^2$
$x = \pm\sqrt{7}$

18. $0 = -16x^2 + 120$

$16x^2 = 120$
$x^2 = \frac{15}{2}$
$x = \pm\sqrt{\frac{15}{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\pm\sqrt{30}}{2}$

19. $(2x - 3)^2 = 25$

$2x - 3 = \pm 5$
$2x = 3 \pm 5$
$2x = 3+5$ $\quad 2x = 3-5$
$2x = 8$ $\quad 2x = -2$
$x = 4$ $\quad x = -1$

20. $3(x - 2)^2 + 4 = 52$

$3(x-2)^2 = 48$
$(x-2)^2 = 16$
$x - 2 = \pm 4$
$x = 2+4 \quad 2-4$
$x = \{6, -2\}$

21. $3(x^2 - 1) = 9$

$(x^2 - 1) = 3$
$x^2 = 4$
$x = \pm 2$

22. $-3(x + 2)^2 = 48$

$(x+2)^2 = -16$
$x + 2 = \pm 4i$
$x = 2 \pm 4i$