

Solving Trigonometric Equations

Name Master E
Date _____ Block _____

The process of solving trigonometric equations is very similar to solving algebraic equations. In a trigonometric equation, however, we are looking for all values of the angle that will make the equation into a true statement. Keep in mind that there may be more than one angle that can make the equation true.

Example A: Solve the equation for θ if $0^\circ \leq \theta < 360^\circ$:

$$\begin{aligned} 2 \cos \theta - 1 &= 0 \\ 2 \cos \theta &= 1 \\ \cos \theta &= \frac{1}{2} \\ \theta &= \cos^{-1}\left(\frac{1}{2}\right) \\ \theta &= 60^\circ \text{ or } 300^\circ \end{aligned}$$

Example B: Solve the equation for t if $0 \leq t < 2\pi$:

$$\begin{aligned} 2 \cos t - \sqrt{2} &= 0 \\ 2 \cos t &= \sqrt{2} \\ \cos t &= \frac{\sqrt{2}}{2} \\ t &= \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) \\ t &= \frac{\pi}{4} \text{ or } \frac{7\pi}{4} \end{aligned}$$

Solve each equation for θ if $0^\circ \leq \theta < 360^\circ$.

1. $2 \sin \theta = 1$

$$\begin{aligned} \sin \theta &= \frac{1}{2} \\ \theta &= \sin^{-1}\left(\frac{1}{2}\right) \\ \theta &= 30^\circ \text{ \& } 150^\circ \end{aligned}$$

Solve each equation for t if $0 \leq t < 2\pi$.

2. $4 \sin t - \sqrt{3} = 2 \sin t$

$$\begin{aligned} 2 \sin t &= \sqrt{3} \\ \sin t &= \frac{\sqrt{3}}{2} \\ t &= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ t &= \frac{\pi}{3} \text{ \& } \frac{2\pi}{3} \end{aligned}$$

3. $2 \cos \theta - \sqrt{3} = 0$

$$\begin{aligned} 2 \cos \theta &= \sqrt{3} \\ \cos \theta &= \frac{\sqrt{3}}{2} \\ \theta &= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ \theta &= 30^\circ \text{ \& } 330^\circ \end{aligned}$$

4. $2 \cos t = 6 \cos t - \sqrt{12}$

$$\begin{aligned} -4 \cos t &= -\sqrt{12} \\ \cos t &= \frac{\sqrt{12}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \\ t &= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ t &= \frac{\pi}{6} \text{ \& } \frac{11\pi}{6} \end{aligned}$$

5. $2 \tan \theta + 2 = 0$

$$\begin{aligned} 2 \tan \theta &= -2 \\ \tan \theta &= -1 \\ \theta &= \tan^{-1}(-1) \\ \theta &= 135^\circ \text{ \& } 315^\circ \end{aligned}$$

6. $3 \sin t + 5 = -2 \sin t$

$$\begin{aligned} 5 \sin t &= -5 \\ \sin t &= -1 \\ t &= \sin^{-1}(-1) \\ t &= \frac{3\pi}{2} \end{aligned}$$

7. $3 \cot \theta + 3 = 0$

$$\begin{aligned} 3 \cot \theta &= -3 \\ \cot \theta &= -1 \\ \theta &= \cot^{-1}(-1) = \tan^{-1}(-1) \\ \theta &= 135^\circ \text{ \& } 315^\circ \end{aligned}$$

8. $2\sqrt{3} \cos t + 7 = 4$

$$\begin{aligned} 2\sqrt{3} \cos t &= -3 \\ \cos t &= \frac{-3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2} \\ t &= \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) \\ t &= \frac{5\pi}{6} \text{ \& } \frac{7\pi}{6} \end{aligned}$$

9. $\sqrt{2} \sec \theta = 2$

$\sec \theta = \frac{2}{\sqrt{2}}$

$\cos \theta = \frac{\sqrt{2}}{2}$

$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\theta = 45^\circ \text{ \& } 315^\circ$

10. $2 \sin t + 3 = 3$

$2 \sin t = 0$

$\sin t = 0$

$t = \sin^{-1}(0)$

$t = 0 \text{ \& } \pi$

11. $2\sqrt{3} \csc \theta + 5 = 9$

$2\sqrt{3} \csc \theta = 4$

$\csc \theta = \frac{4}{2\sqrt{3}}$

$\sin \theta = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\theta = 60^\circ \text{ \& } 120^\circ$

12. $6 \cos t - \sqrt{3} = 4 \cos t$

$-\sqrt{3} = -2 \cos t$

$\frac{\sqrt{3}}{2} = \cos t$

$t = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$t = \frac{\pi}{6} \text{ \& } \frac{11\pi}{6}$

Example C: Find all solutions in the interval $0^\circ \leq \theta < 360^\circ$. Use a calculator on the last step and write the answer to the nearest tenth of a degree.

$4 \sin \theta - 3 = 0$

$4 \sin \theta = 3$

$\sin \theta = \frac{3}{4}$

$\theta = \sin^{-1}\left(\frac{3}{4}\right)$ Use the calculator to compute (DEGREE MODE)

$\theta = 48.6^\circ$

*sin is positive in QI and QII. You must also find the angle in QII.

$\theta = 48.6^\circ$ or 131.4° Calculate this using a 48.6° reference angle (subtract 48.6° from 180°).

Find all solutions in the interval $0^\circ \leq \theta < 360^\circ$.

13. $\sin \theta - 3 = 5 \sin \theta$

$-3 = 4 \sin \theta$

$\sin \theta = -\frac{3}{4}$

sin is - in Q3 & Q4

$\theta = \sin^{-1}\left(-\frac{3}{4}\right)$



$\theta = 180 + 48.6$ and $360 - 48.6$
 $\theta = 228.6^\circ$ and 311.4°

14. $3 \cos \theta - 7 = 4 \cos \theta - 4$

$-3 = \cos \theta$

$\theta = \cos^{-1}(-3)$

Range is $[-1, 1]$!

No solution

15. $2 \cos \theta - 5 = 3 \cos \theta - 2$

$-3 = \cos \theta$

$\theta = \cos^{-1}(-3)$

No solution

16. $\sin \theta - 5 = -2 \sin \theta$

$3 \sin \theta = 5$

$\sin \theta = \frac{5}{3}$

$\theta = \sin^{-1}\left(\frac{5}{3}\right)$

Range is $[-1, 1]$
 $\frac{5}{3} > 1$!

No solution