

Solving Trigonometric Equations using Identities

Name _____ Date _____ Block _____

Solve for all values of x : $0^\circ \leq x \leq 360^\circ$

1. $4\cos^2 x = 1$

$$\cos^2 x = \frac{1}{4}$$

$$\cos x = \pm \frac{1}{2}$$

$$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

2. $\cos x \tan x - \sin^2 x = 0$

$$\cos x \cdot \frac{\sin x}{\cos x} - \sin^2 x = 0$$

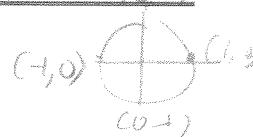
$$\sin x - \sin^2 x = 0$$

$$\sin x(1 - \sin x) = 0$$

$$\sin x = 0 \quad 1 - \sin x = 0$$

$$x = 0^\circ, 180^\circ, 360^\circ \quad x = 90^\circ$$

$$x = 0^\circ, 90^\circ, 180^\circ, 360^\circ$$



3. $\sin^2 x - \sin x = 0$

$$\sin x(\sin x - 1) = 0$$

$$\sin x = 0 \quad \sin x = 1$$

Same as #2

4. $2\sin^2 x - \sin x = 1$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \quad \sin x = 1$$

$$x = 210^\circ, 330^\circ \quad 90^\circ$$

$$x = 90^\circ, 210^\circ, 330^\circ$$

5. $\cot^2 x = 1$

$$\cot x = \pm 1$$

$$\tan x = \pm 1$$

$$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

6. $\tan^2 x + \sec x = 1$

$$\sec^2 x - 1 + \sec x = 1$$

$$\sec^2 x + \sec x - 2 = 0$$

$$(\sec x + 2)(\sec x - 1) = 0$$

$$\sec x + 2 = 0 \quad \sec x - 1 = 0$$

$$\sec x = -2 \quad \sec x = 1$$

$$\cos x = -\frac{1}{2} \quad \cos x = 1$$

$$x = 120^\circ, 240^\circ \quad 0^\circ, 360^\circ$$

$$x = 0^\circ, 120^\circ, 240^\circ, 360^\circ$$

Solve for all values of θ : $0^\circ \leq \theta \leq 2\pi$

7. $\sin^2 \theta \cos \theta = \cos \theta$ typo

$$\begin{aligned} \sin^2 \theta \cos \theta - \cos \theta &= 0 \\ \cos \theta (\sin^2 \theta - 1) &= 0 \\ \cos \theta = 0 \quad \sin^2 \theta - 1 &= 0 \\ \sin^2 \theta &= 1 \\ \sin \theta &= \pm 1 \end{aligned}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

8. $\csc^2 \theta - 3\csc \theta + 2 = 0$

$$(\csc \theta - 2)(\csc \theta - 1) = 0$$

$$\csc \theta = 2 \quad \csc \theta = 1$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = 1$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

9. $\cot \theta = \cot^3 \theta$

$$\begin{aligned} 0 &= \cot^3 \theta - \cot \theta \\ 0 &= \cot \theta (\cot^2 \theta - 1) \\ \cot \theta = 0 &\quad \cot^2 \theta = 1 \\ \tan \theta = 0 &\quad \cot \theta \neq 0 \\ \frac{\pi}{2}, \frac{3\pi}{2} &\quad \tan \theta = \pm 1 \\ &\quad \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \end{aligned}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

11. $4\sin^2 \theta - 1 = 0$

$$\begin{aligned} 4\sin^2 \theta &= 1 \\ \sin^2 \theta &= \frac{1}{4} \\ \sin \theta &= \pm \frac{1}{2} \end{aligned}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

10. $\sqrt{2} \sin^3 \theta = \sin^2 \theta$

$$\begin{aligned} \sqrt{2} \sin^3 \theta - \sin^2 \theta &= 0 \\ \sin^2 \theta (\sqrt{2} \sin \theta - 1) &= 0 \\ \sin^2 \theta &= 0 \quad \sin \theta = \frac{1}{\sqrt{2}} = \frac{\pi}{4} \\ \sin \theta &= 0 \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4} \\ \theta &= 0, \pi, 2\pi \end{aligned}$$

$$\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}$$

12. $\tan \theta = \sin \theta$

$$\frac{\sin \theta}{\cos \theta} - \sin \theta = 0$$

$$\frac{\sin \theta - \sin \theta \cos \theta}{\cos \theta} = 0$$

$$\sin \theta (1 - \cos \theta) = 0$$

$$\sin \theta = 0 \quad 1 - \cos \theta = 0$$

$$\theta = 0, \pi, 2\pi \quad \theta = 0, 2\pi$$

$$\theta = 0, \pi, \frac{3\pi}{2}$$