The Number e

Name KEY

Date ______________ Block ____

Suppose you invest \$1000 in a bank that pays a rate of 9%, compounded n times per year. Find the amount of money you would have in the bank if the interest were compounded various times per year. *Fill in the chart.*

Compound Interest formula: $A = P\left(1+\frac{r}{n}\right)^{nt}$

Pe^{rt}

$\boxed{1000e^{.09(1)}}$

9% Interest Compounded:	n	Amount of money accumulated After 1 year in the bank:
Annually	1	$A = 1000\left(1+\frac{.09}{1}\right)^{(1\cdot 1)}$ = \$ 1090
Semi-annually	2	\$ 1092.03
Quarterly	4	\$ 1093.08
Monthly	12	\$ 1093.81
Semi-monthly	24	\$ 1093.99
Weekly	52	\$ 1094.09
Daily	365	\$ 1094.16
Hourly	8760	\$ 1094.17
Every minute	525,600	\$ 1094.17
Every second	31,536,000	\$ 1094.17

As n approaches infinity, the amount of money earned approaches a fixed value. Interest in this case is said to be compounded continuously. If we let P = 1, r = 1, and t = 1 in the compound interest formula, we get:

n	$\left(1+\frac{1}{n}\right)^n$
1	2.00000000
10	2.59374246
100	2.70481383
1,000	2.71692393
10,000	2.71814593
100,000	2.71826824
1,000,000	2.71828047
10,000,000	2.71828169
100,000,000	2.71828181
1,000,000,000	2.71828183

$e \approx 2.$

The natural base "e" is irrational and is defined as follows: as $n \to +\infty$,

$$n \to \left(1+\frac{1}{n}\right)^n \to e \approx 2.71828183$$

Euler discovered "e" (1707 – 1783)
It is one of the significant discoveries in math such as pi and imaginary numbers

Continuously Compounded Interest:

$A = Pe^{rt}$

Use when money is compounded continuously.

THINK: Pert shampoo works continuously to make your hair luxurious!

A: Amount of money generated after t years

P: Principal amount

e $\approx$ 2.718281828459 (use the e on your TI-83!)

r: rate (% increase or decrease - always convert to a decimal)

t: time (in terms of the number of years)

Solve each word problem. Show all work!

1. If $200 is invested and earns 8.0% simple interest, what is the final value of the investment after 6 years?

Simple interest

$$200(1+.08)^{6} = 200(1.08)^{6} = \boxed{\$317.37}$$

2. If $4000 is invested at an annual rate of 6.0% compounded monthly, what will be the final value of the investment after 10 years? What if it was invested at an annual rate of 6.0% compounded continuously?

Comp. Int.

$$A = 4000\left(1+\frac{.06}{12}\right)^{(12\cdot 10)} = \boxed{\$7,277.59}$$

$$= 4000e^{(.06\cdot 10)}$$

$$\boxed{\$7288.48}$$

3. Suppose that you are going to need $10,000 in thirty-six months when your child starts attending college. You want to invest in a bank that will yield 3.5% interest, compounded monthly. How much should you invest in the bank?

= 3 yr

Comp. int.

$$10,000 = P\left(1+\frac{.035}{12}\right)^{(12\cdot 3)}$$

in years!

$$10,000 = P(1.1105\ldots)$$

$$P = \frac{10,000}{1.1105} = \boxed{\$9,004.62}$$

4. Certain bacteria, given favorable growth conditions, grow continuously at a rate of 4.6% a day. Find the bacterial population after thirty-six hours, if the initial population was 250 bacteria.

Cont. comp.

$$A = Pe^{rt}$$

$$A = 250e^{.046t} = 250e^{.046(1.5)} = \boxed{267.86}$$

36 HRS = 1.5 days
compounded daily

5. What will be the amount yielded if you invest $20,500 for 15 years at an annual rate of 7.5%, compounded semiannually?

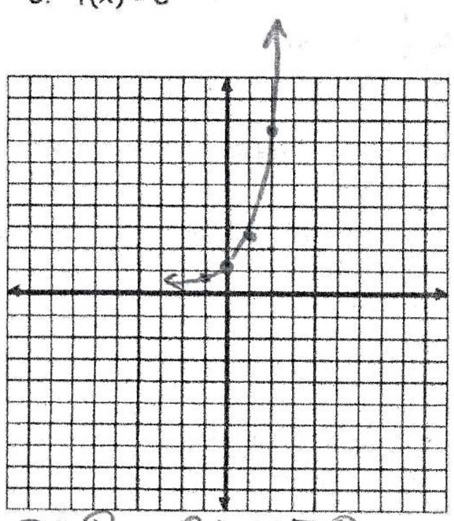
Comp. Int.

$$20,500\left(1+\frac{.075}{2}\right)^{(2\cdot 15)} = \boxed{\$61,858.16}$$

Graph each function

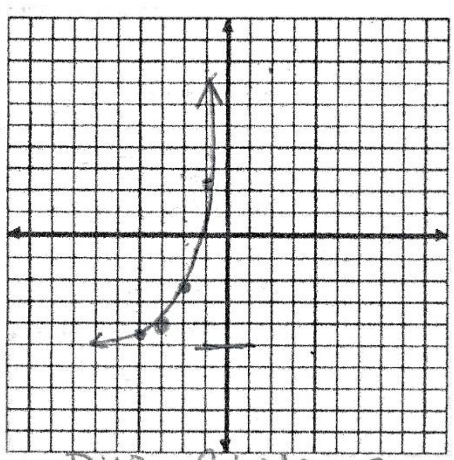
shift #6 ←3 ↓5

6. $f(x) = e^{x}$



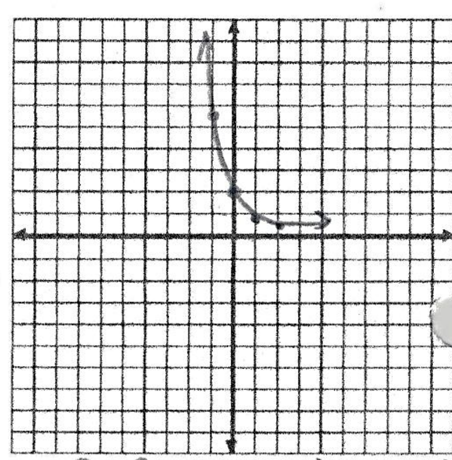
D: ℝ R: $y>0$
Asymp: $y=0$

7. $f(x) = (e)^{x+3} - 5$



D: ℝ R: $y>-5$
Asymp: $y=-5$

8. $f(x) = 2(e)^{-x} = 2\left(\frac{1}{e}\right)^{x}$



D: ℝ R: $y>0$ Asymp: $y=0$