

Trigonometric Equations Involving Factoring

Name Master E
Date _____ Block _____

The process of solving some trigonometric equations can be compared to solving algebraic quadratic equations. In a quadratic equation, we are supposed to factor the equation to find a solution or solutions.

Factoring Example:

Solve the equation for x : $2x^2 + 7x = 4$

First, you must put the equation in standard form:

$$2x^2 + 7x - 4 = 0$$

Try to factor the left side of the equation:

$$(2x - 1)(x + 4) = 0$$

Set each factor equal to zero and solve:

$$2x - 1 = 0 \quad \text{or} \quad x + 4 = 0$$

$$2x = 1 \qquad \qquad x = -4$$

$$x = \frac{1}{2}$$

The solutions are $x = \frac{1}{2}$ or $x = -4$.

Trigonometry Example using Factoring:

Solve $2 \sin^2 t + 7 \sin t = 4$, if $0 \leq t < 2\pi$.

Notice that this example is just like above, except $\sin t$ is in place of the x . Solve in a similar way.

First, you must put the equation in standard form:

$$2 \sin^2 t + 7 \sin t - 4 = 0$$

Try to factor the left side of the equation:

$$(2 \sin t - 1)(\sin t + 4) = 0$$

Set each factor equal to zero and solve:

$$2 \sin t - 1 = 0 \quad \text{or} \quad \sin t + 4 = 0$$

$$2 \sin t = 1 \qquad \qquad \sin t = -4$$

$$\sin t = \frac{1}{2}$$

$$t = \sin^{-1}(-4)$$

$$t = \sin^{-1}\left(\frac{1}{2}\right)$$

no solution

$$t = \frac{\pi}{6}, \frac{5\pi}{6}$$

Thus, the solutions are $t = \frac{\pi}{6}, \frac{5\pi}{6}$.

Solve each equation for θ if $0^\circ \leq \theta < 360^\circ$.

1. $(\cos \theta + 1)(2 \cos \theta - 1) = 0$

$$\cos \theta + 1 = 0$$

$$2 \cos \theta - 1 = 0$$

$$\cos \theta = -1$$

$$2 \cos \theta = 1$$

$$\theta = 180^\circ$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = 60^\circ \text{ \& } 300^\circ$$

$$\therefore \theta = 60^\circ, 180^\circ, \text{ \& } 300^\circ$$

2. $\cos \theta + 2 \sin \theta \cos \theta = 0$

$$\cos \theta (1 + 2 \sin \theta) = 0$$

$$\cos \theta = 0 \quad 1 + 2 \sin \theta = 0$$

$$\theta = 90^\circ \text{ \& } 270^\circ$$

$$2 \sin \theta = -1$$

$$\sin \theta = -\frac{1}{2}$$

$$\theta = 210^\circ \text{ \& } 330^\circ$$

$$\therefore \theta = 90^\circ, 210^\circ, 270^\circ, \text{ \& } 330^\circ$$

$$3. 2 \cos^2 \theta + 5 \cos \theta - 3 = 0$$

$$(2 \cos \theta - 1)(\cos \theta + 3) = 0$$

$$2 \cos \theta - 1 = 0 \quad \cos \theta + 3 = 0$$

$$2 \cos \theta = 1 \quad \cos \theta = -3$$

$$\cos \theta = \frac{1}{2} \quad \theta = \emptyset$$

$$\theta = 60^\circ \text{ \& } 300^\circ$$

$$\therefore \theta = 60^\circ \text{ \& } 300^\circ$$

$$5. \tan \theta (\tan \theta + 1) = 0$$

$$\tan \theta = 0 \quad \tan \theta + 1 = 0$$

$$\tan \theta = -1$$

$$\theta = 0^\circ \text{ \& } 180^\circ$$

$$\theta = 135^\circ \text{ \& } 315^\circ$$

$$\therefore \theta = 0^\circ, 135^\circ, 180^\circ, \text{ \& } 315^\circ$$

$$7. 2 \cos^2 \theta + \cos \theta - 1 = 0$$

$$(2 \cos \theta - 1)(\cos \theta + 1) = 0$$

$$2 \cos \theta - 1 = 0 \quad \cos \theta + 1 = 0$$

$$2 \cos \theta = 1 \quad \cos \theta = -1$$

$$\cos \theta = \frac{1}{2} \quad \theta = 180^\circ$$

$$\theta = 60^\circ \text{ \& } 300^\circ$$

$$\therefore \theta = 60^\circ, 180^\circ, \text{ \& } 300^\circ$$

$$9. 2 \sin^2 \theta - 7 \sin \theta = -3$$

$$2 \sin^2 \theta - 7 \sin \theta + 3 = 0$$

$$(2 \sin \theta - 1)(\sin \theta - 3) = 0$$

$$2 \sin \theta - 1 = 0 \quad \sin \theta - 3 = 0$$

$$2 \sin \theta = 1 \quad \sin \theta = 3$$

$$\sin \theta = \frac{1}{2} \quad \theta = \emptyset$$

$$\theta = 30^\circ \text{ \& } 150^\circ$$

$$\therefore \theta = 30^\circ \text{ \& } 150^\circ$$

$$4. (\cos \theta - 1)(2 \cos \theta + 1) = 0$$

$$\cos \theta - 1 = 0 \quad 2 \cos \theta + 1 = 0$$

$$\cos \theta = 1 \quad 2 \cos \theta = -1$$

$$\theta = 0^\circ$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = 120^\circ \text{ \& } 240^\circ$$

$$\therefore \theta = 0^\circ, 120^\circ, \text{ \& } 240^\circ$$

$$6. \cos \theta - 2 \sin \theta \cos \theta = 0$$

$$\cos \theta (1 - 2 \sin \theta) = 0$$

$$\cos \theta = 0 \quad 1 - 2 \sin \theta = 0$$

$$\theta = 90^\circ \text{ \& } 270^\circ \quad -2 \sin \theta = -1$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = 30^\circ \text{ \& } 150^\circ$$

$$\therefore \theta = 30^\circ, 90^\circ, 150^\circ, \text{ \& } 270^\circ$$

$$8. (2 \sin \theta - \sqrt{3})(2 \sin \theta - 1) = 0$$

$$2 \sin \theta - \sqrt{3} = 0 \quad 2 \sin \theta - 1 = 0$$

$$2 \sin \theta = \sqrt{3}$$

$$2 \sin \theta = 1$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = 60^\circ \text{ \& } 120^\circ$$

$$\theta = 30^\circ \text{ \& } 150^\circ$$

$$\therefore \theta = 30^\circ, 60^\circ, 120^\circ, \text{ \& } 150^\circ$$

$$10. \sin \theta \cos \theta - \frac{1}{2} \cos \theta = 0$$

$$\cos \theta (\sin \theta - \frac{1}{2}) = 0$$

$$\cos \theta = 0 \quad \sin \theta - \frac{1}{2} = 0$$

$$\theta = 90^\circ \text{ \& } 270^\circ \quad \sin \theta = \frac{1}{2}$$

$$\theta = 30^\circ \text{ \& } 150^\circ$$

$$\therefore \theta = 30^\circ, 90^\circ, 150^\circ, \text{ \& } 270^\circ$$