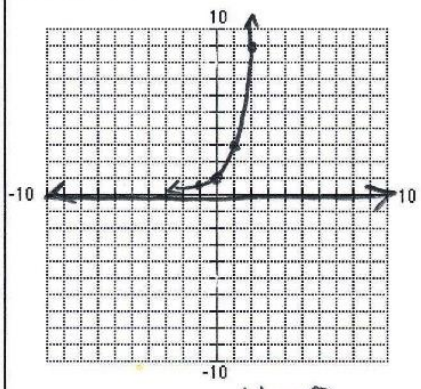
Algebra 2 Trigonometry Unit 7 Test Review
Exponential and Logarithmic Functions

Name Master Ed
Date _____ Block _____

- *There will be two parts to the test: A NON-CALCULATOR portion and a CALCULATOR portion!*
- *Know how to graph exponential functions and logarithmic functions. Be able to determine the domain and range, the y-intercept, and the equations of the asymptotes of these functions.*

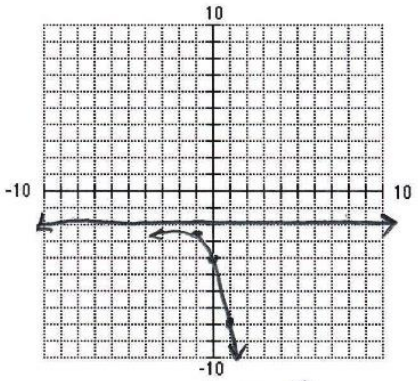
Graph each function without a calculator. State the y-intercept, the domain and range using interval notation, and the equation of the asymptote.

1. $y = 3^x$



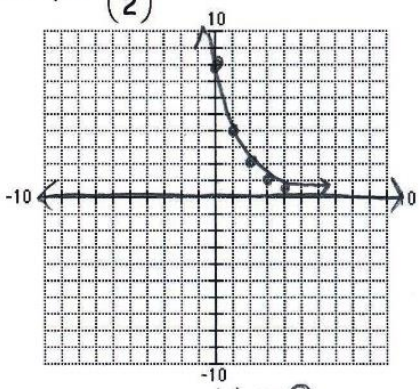
Asymptote: $y=0$
y-intercept: $(0,1)$
Domain: $(-\infty,\infty)$
Range: $(0,\infty)$

2. $y = -2(3)^x - 2$ $(-1,-2\frac{2}{3})$



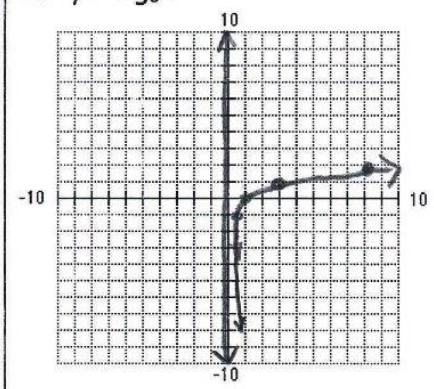
Asymptote: $y=-2$
y-intercept: $(0,-4)$
Domain: $(-\infty,\infty)$
Range: $(-\infty,-2)$

3. $y = \left(\frac{1}{2}\right)^{x-3}$



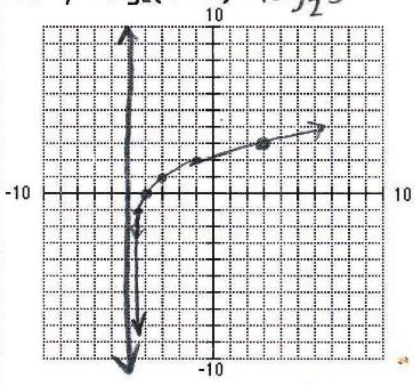
Asymptote: $y=0$
y-intercept: $(0,8)$
Domain: $(-\infty,\infty)$
Range: $(0,\infty)$

4. $y = \log_3 x$



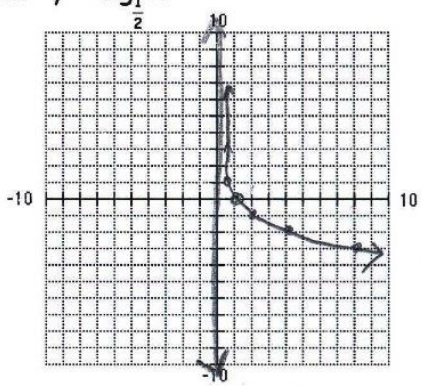
Asymptote: $x=0$
y-intercept: NONE
Domain: $(0,\infty)$
Range: $(-\infty,\infty)$

5. $y = \log_2(x+5)$ $\log_2 5$



Asymptote: $x=-5$
y-intercept: $\approx(0,2.\ldots)$
Domain: $(-5,\infty)$
Range: $(-\infty,\infty)$

6. $y = \log_{\frac{1}{2}} x$



Asymptote: $x=0$
y-intercept: NONE
Domain: $(0,\infty)$
Range: $(-\infty,\infty)$

- *Know how to determine whether a function is a growth or decay function.*

State whether the function represents exponential growth or exponential decay. (NO calculator)

7. $f(x) = 5\left(\frac{3}{4}\right)^x$	8. $f(x) = 2e^x$	9. $f(x) = 3(6)^{-x}$	10. $f(x) = 4(3)^x$	11. $f(x) = 2e^{-3x}$
Decay	Growth	Decay	Growth	Decay

◆ *Know how to change an expression from exponential form to logarithmic form and vice versa.*

Rewrite each equation in exponential form. (NO calculator)

12. $\log_5 \frac{1}{5} = -1$	13. $\log_8 512 = 3$	14. $\log_{14} 196 = 2$	15. $\log_{105} 11{,}025 = 2$
$5^{-1} = \frac{1}{5}$	$8^3 = 512$	$14^2 = 196$	$105^2 = 11{,}025$

Rewrite each equation in logarithmic form. (NO calculator)

16. $2^5 = 32$	17. $10^{-1} = 0.1$	18. $\left(\frac{1}{2}\right)^{-1} = 2$	19. $36^{-\frac{1}{2}} = \frac{1}{6}$
$\log_2 32 = 5$	$\log 0.1 = -1$	$\log_{\frac{1}{2}} 2 = -1$	$\log_{36} \frac{1}{6} = -\frac{1}{2}$

Know how to simplify a logarithm without a calculator.

Evaluate each expression without using a calculator.

20. $\log_2 16$	21. $\log_5 25$	22. $\log_{11} 1$	23. $\log_{\frac{1}{4}} 2$
4	2	0	$-\frac{1}{2}$
24. $\log_3 3^{-2.27}$	**25.** $\log_7 343$	**26.** $\log_{29} 29$	**27.** $\log_9 9^3$
-2.27	3	1	3

◆ *Know how to find the inverse of a function. (Remember that a logarithmic function is the inverse of an exponential function).*

Find the inverse of each function. (NO calculator)

28. $f(x) = \log_{\frac{1}{3}} x$	29. $y = \ln(x-3)$	30. $f(x) = 7^x$
$x = \log_{\frac{1}{3}} y$	$x = \ln(y-3)$	$x = 7^y$
$f^{-1}(x) = \left(\frac{1}{3}\right)^x$	$e^x = y-3 \Rightarrow e^x + 3 = y$	$\log_7 x = y$
	$f^{-1}(x) = e^x + 3$	$f^{-1}(x) = \log_7 x$

◆ *Know how to expand and condense a logarithmic expression. KNOW THE PROPERTIES!*

Expand each expression. (NO calculator)

31. $\log_3 9x$	32. $\log 3x^4$	33. $\log_6 x^5$	34. $\ln 15x$
$2 + \log_3 x$	$\log 3 + 4\log x$	$5\log_6 x$	$\ln 15 + \ln x$
$\log_3 9 + \log_3 x$			
35. $\log_7 49x^2$	**36.** $\log \sqrt{9x}$	**37.** $\ln x^{\frac{1}{3}} y^4$	**38.** $\log x^2 y^3 z^4$
$2 + 2\log_7 x$	$\log 9^{\frac{1}{2}} + \log x^{\frac{1}{2}}$	$\frac{1}{3}\ln x + 4\ln y$	$2\log x + 3\log y + 4\log z$
$\log_7 49 + 2\log_7 x$	$\log 3 + \frac{1}{2}\log x$		

Condense each expression. (NO calculator)

39. $\log_4 7 + \log_4 10 - \log_4 2$	40. $4\ln x + 6\ln y + 3\ln z$	41. $5\log_4 3 + 6\log_4 x + 7\log_4 y$
$\log_4 \frac{70}{2} = \log_4 35$	$\ln x^4 + \ln y^6 + \ln z^3$	$\log_4 3^5 + \log_4 x^6 + \log_4 y^7$
	$\ln x^4 y^6 z^3$	$\log_4 243 x^6 y^7$

42. $\frac{1}{4}(\ln 9 - \ln x) + \frac{1}{4}\ln 3$	43. $3(\log_5 10 - \log_5 2) + \frac{1}{2}\log_5 \frac{1}{100}$
$\ln 9^{\frac{1}{4}} - \ln x^{\frac{1}{4}} + \ln 3^{\frac{1}{4}}$	$3\left(\log_5 \frac{10}{2}\right) + \log_5 \left(\frac{1}{100}\right)^{\frac{1}{2}}$
$\ln \frac{\sqrt[4]{9 \cdot 3}}{\sqrt[4]{x}} \cdot \frac{\sqrt[4]{x^3}}{\sqrt[4]{x^3}} = \ln \frac{\sqrt[4]{27x^3}}{x}$	$3 + \log_5 \frac{1}{10}$

◆ *Know how to evaluate an expression by applying the properties of logarithms.*

Use a property of logarithms to evaluate each expression. (NO calculator)

44. $\log_2 (4 \cdot 8)$
$\log_2 4 + \log_2 8$
$2 + 3 = 5$

45. $\ln e^3$
$3 \ln e =$
$3(1) = 3$

46. $\log_2 8^2$
$2 \log_2 8 = 2(3) = 6$

47. $\log_6 216$
$\log_6 6^3 = 3 \log_6 6 =$
3

48. $\log \frac{1}{100}$
$\log 1 - \log 100$
$0 - 2 = -2$

49. $\ln \frac{1}{e^5}$
$\ln 1 - \ln e^5$
$0 - 5 = -5$

50. $\log 0.001$
$\log \frac{1}{1000} = \log 1 - \log 1000$
$0 - 3 = -3$

51. $\log_3 27^2$
$2 \log_3 27 = 2(3)$
6

◆ *Know how to evaluate an expression using the CHANGE-OF-BASE FORMULA.*

Use the change-of-base formula to evaluate each expression. (May use calculator)

52. $\log_6 24$
$\frac{\log 24}{\log 6} = 1.774$

53. $\log_9 \frac{5}{16}$
$\frac{\log \frac{5}{16}}{\log 9} -0.529$

54. $\log_2 12$
$\frac{\log 12}{\log 2} = 3.585$

◆ *Know how to solve a logarithmic or exponential equation.*

Solve each equation. Check for extraneous solutions. (May use calculator)

55. $4.7^x = 32$
$\log_{4.7} 32 = x$
$\frac{\log 32}{\log 4.7} = x$
$2.239 \approx x$

56. $4^x - 3 = 11$
$4^x = 14$
$\log_4 14 = x$
$\frac{\log 14}{\log 4} = x$
$1.904 \approx x$

57. $3^{x+2} = 9^{x+1}$
$3^{x+2} = 3^{2(x+1)}$
$x + 2 = 2x + 2$
$0 = x$

58. $\log_5 (2x + 10) = \log_5 4x$
$2x + 10 = 4x$
$10 = 2x$
$5 = x$

59. $\ln (5 - x) = 12$
$e^{12} = 5 - x$
$e^{12} - 5 = -x$
$-(e^{12} - 5) = x$
$-162749.791 \approx x$

60. $\log_2 x + \log_2 (x + 4) = 5$
$\log_2 x^2 + 4x = 5$
$2^5 = x^2 + 4x$
$0 = x^2 + 4x - 32$
$(x + 8)(x - 4)$
$x = -8, x = 4$
$x = 4$

64
16

61. $\ln 8x = 4$
$e^4 = 8x$
$\frac{e^4}{8} = x$
$6.825 \approx x$

62. $9000 = 500(1.065)^x$
$18 = 1.065^x$
$\log_{1.065} 18 = x$
$\frac{\log 18}{\log 1.065} = x$
$45.897 \approx x$

63. $3^{x-2} = 5^{2x}$
$\frac{(x-2)(\log 3)}{\log 3} = \frac{(2x)(\log 5)}{\log 3}$
$x - 2 = 2.9299x$
$-2 = 1.9299x$
$-1.036 \approx x$

- *Know how to solve a growth problem.* $y = a(1 + r)^t$ *Know what is meant by "growth factor".*
- *Know how to solve a decay problem.* $y = a(1 - r)^t$ *Know what is meant by "decay factor".*
- *Know how to calculate compounded interest:* $A = P\left(1+\frac{r}{n}\right)^{nt}$ *& continuously compounded interest:* $A = Pe^{rt}$

Solve each problem. (May use a calculator)

64. Carl plans to invest $500 at 8.25% interest, compounded continuously. How long will it take for his money to triple?

$1500 = 500e^{(.0825 \cdot t)}$

$3 = e^{.0825t}$

$\ln 3 = .0825t$

$t = \frac{\ln 3}{.0825}$

13.317 years

≈ 13 years 3.8 months

65. A piece of machinery valued at $250,000 depreciates at a steady rate of 12% per year. After how many years will the value have depreciated to $100,000?

$100{,}000 = 250{,}000(1-.12)^t$

$.4 = .88^t$

$\log .4 = t \log .88$

$t = \frac{\log .4}{\log .88}$

7.168 years

≈ 7 years 2 months

66. Ray invested $10,000 in an account which yields 4.5% interest compounded monthly. Assuming no deposits or withdrawals are made, what will the balance of the account be after 5 years?

$A = 10{,}000\left(1+\frac{.045}{12}\right)^{(12 \cdot 5)} =$ $12517.96

67. Dave bought a new car 8 years ago for $5400. To buy a new car comparably equipped now would cost $12,500. Assuming a steady rate of increase what was the yearly rate of inflation in car prices over the 8 year period?

$12{,}500 = 5400(1+r)^8$

$(2.315)^{\frac{1}{8}} = ((1+r)^8)^{\frac{1}{8}}$

$1.1106 = 1+r$

$.1106 = -r$

$-0.1106 \approx r$

$r = 11.06\%$

68. An organism of a certain type can grow continuously from 30 to 195 organisms in 5 hours. Find k, the rate of continuous growth, for the growth formula ($y = ne^{kt}$), where t is the time in hours.

$195 = 30e^{k \cdot 5}$

$6.5 = e^{5k}$

$\ln 6.5 = 5k$

$k = \frac{\ln 6.5}{5} = .37436 =$ 37.44%

69. An equation for loudness L in decibels is given by L = 10log R, where R is the sound's relative intensity. An air-raid siren can reach 150 decibels and jet engine noise can reach 120 decibels. How many times greater is the relative intensity of the air-raid siren than that of the jet engine noise?

$150 = 10 \log R$

$15 = \log R$

$10^{15} = R$

$120 = 10 \log R$

$12 = \log R$

$10^{12} = R$

$\frac{10^{15}}{10^{12}} = 10^3 =$ 1,000 times greater