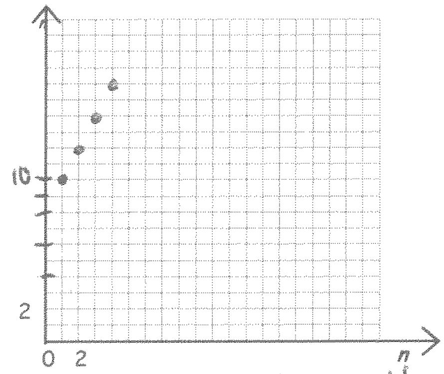
Algebra 2 & Trigonometry Test Review — Name: Master Copy

UNIT 7: Sequences & Series — Date ______ Block ______

Learning Target A	I can identify, mathematically define, and graph arithmetic, geometric, and infinite geometric sequences and series, generalize patterns in a sequence using explicit and recursive formulas.

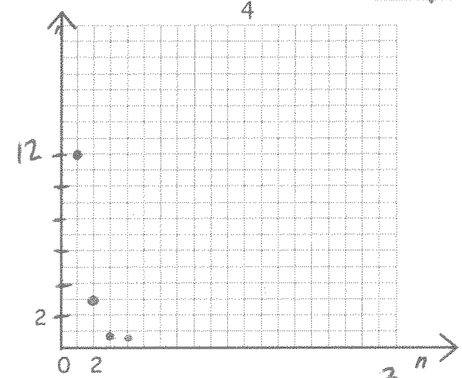
1-2: State whether the sequence generated by the recursive formula is arithmetic or geometric. Then write the first four terms of the sequence and sketch a graph below your terms.

1. $a_1 = 10, \quad a_n = a_{n-1} + 2, n > 2$ Arithmetic



10, 12, 14, 16

2. $a_1 = 12, a_n = \frac{1}{4} a_{n-1}, n > 2$ Geometric



12, 3, $\frac{3}{4}$, $\frac{3}{16}$

3-4: Write the recursive and explicit formula that generates each sequence, and then find the indicated term.

3. -5, -10, -15, -20, . . .

Recursive: $a_1 = -5, a_n = a_{n-1} - 5$, where $n \geq 2$

Explicit: $a_n = -5n$

$a_{10} =$ $-5(10)$ or $-5 + 9(-5)$ = -50

4.

n	1	2	3	4	5
a_n	-2	6	-18	54	-162

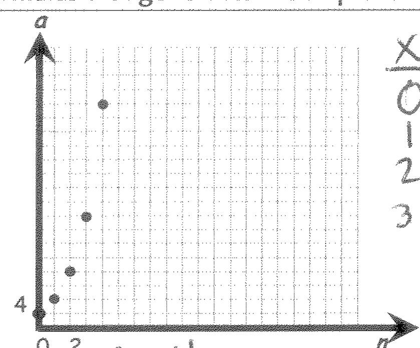
Recursive: $a_1 = -2, a_n = -3a_{n-1}$, where $n \geq 2$

Explicit: $a_n = -2(-3)^{n-1}$

$a_{17} =$ $-2(-3)^{16}$ = -86093442

5-6: State whether each indicated sequence is arithmetic or geometric. Then write the recursive and explicit formulas that generates the sequence.

5.



x	y
0	2
1	4
2	8
3	16

Geometric

or $a_1 = 4$ $n \geq 2$

Recursive: $a_0 = 2, a_n = 2a_{n-1}$, where $n \geq 1$

Explicit: $a_n = 4(2)^{n-1}$

6.

n	0	1	2	3	4
a_n	-142	-37	68	173	278

$d = 105$

Arithmetic

Recursive: $a_0 = -142, a_n = a_{n-1} + 105$, where $n \geq 1$

Explicit: $a_n = 105n - 142$

7-8: Find the first four terms of the given sequence.

7. $a_n = 3a_{n-1} - 3, a_1 = 1$

1, 0, -3, -12

$3(1)-3$ $3(0)-3$ $3(-3)-3$

8. $a_n = \frac{1}{2} a_{n-1} + 4, a_1 = 6$

6, 7, 7.5, 7.75

$\frac{1}{2}(6)+4$ $\frac{1}{2}(7)+4$ $\frac{1}{2}(7.5)+4$

Learning Target B	I can find various terms and components of a sequence or series using the appropriate given formulas.

9-12: Identify each sequence as arithmetic, geometric, or neither. Then find the next 3 terms.

9. -5, -3, -1, 1, … $d=2$	10. -4, $-\frac{5}{2}$, -1, $\frac{1}{2}$, … $d=1.5$	11. 12, 6, 3, $\frac{3}{2}$ $r=\frac{1}{2}$	12. $\frac{1}{3}$, -1, 3, -9, … $r=-3$
Arithmetic	Arithmetic	Geometric	Geometric
3, 5, 7	2, 3.5, 5	$\frac{3}{4}$, $\frac{3}{8}$, $\frac{3}{16}$	27, -81, 243

13-16: Find the n^th term in each sequence: (13-14) arithmetic, (15-16) geometric. *Show work in space below.*

13. $a_6 = 17$, $a_{12} = 29$; $a_{30} =$ 65

$17 + 6d = 29$
$6d = 12$
$d = 2$

$a_{30} = 29 + 18(2)$
$a_{12} + 18(d)$

14. $a_1 = 7.5$, $a_9 = -8.5$; $a_6 =$ -2.5

$7.5 + 8d = -8.5$
$8d = -16$
$d = -2$

$a_6 = 7.5 + 5(-2)$
$a_1 + (n-1)(d)$

15. $a_5 = 48$, $a_8 = -384$; $a_{10} =$ -1536

$48r^3 = -384$
$r^3 = -8$
$r = -2$

$a_{10} = 48(-2)^5$
$a_5(r)^{(10-5)}$

16. $a_3 = -6$, $a_6 = \frac{3}{4}$; $a_{10} =$ $\frac{3}{64}$ or .046875

$-6 \cdot r^3 = \frac{3}{4}$
$r^3 = -\frac{1}{8}$
$r = -\frac{1}{2}$

$a_{10} = \frac{3}{4}\left(-\frac{1}{2}\right)^4$
$a_6(r)^{(10-6)}$

17-18: Find the arithmetic means. *Show work (equation) in space below.*

17. $\frac{2}{3}$, 2 ($\frac{6}{3}$), $\frac{10}{3}$, $\frac{14}{3}$, 6 ($\frac{18}{3}$), $\frac{22}{3}$, $\frac{26}{3}$, 10

$2 + 6d = 10$
$6d = 8$
$d = \frac{8}{6} = \frac{4}{3}$

18. $-\frac{484}{3}$, 4 ($\frac{12}{3}$), $\frac{508}{3}$, $\frac{1004}{3}$, 500

$4 + 3d = 500$
$3d = 496$
$d = \frac{496}{3}$

19-20: Find the geometric means. *Show work (equation) in space below.*

19. $\frac{3}{2}$, 3, 6, 12, 24, 48

$3r^3 = 24$
$r^3 = 8$
$r = 2$

20. $\pm\frac{1}{2}$, 2, ± 8, 32, ± 128, 512

$2r^4 = 512$
$r^4 = 256$
$r = \pm 4$

Learning Target C	I can find the sum of a series, including an infinite geometric series, using the appropriate given formulas, and I can use sigma notation to define or interpret a series.

21-22: Write each series in expanded form & find its sum. *Circle your final answer.*

21. $\sum_{n=2}^{7} 2n - 2$

$\underset{(n=2)}{2} + \underset{(n=3)}{4} + \underset{(n=4)}{6} + \underset{(n=5)}{8} + \underset{(n=6)}{10} + \underset{(n=7)}{12} = 42$

22. $\sum_{n=0}^{4} 5(2)^{n-1}$

$\underset{n=0}{\frac{5}{2}} + \underset{n=1}{5} + \underset{n=2}{10} + \underset{n=3}{20} + \underset{n=4}{40} = 77.5$

$\sum dn + a_0$ or $\sum a_1 (r)^{n-1}$

23-28: Express each series in sigma notation. *Remember: use the a_n formula with n as the variable!*

$a_0 = 0$ -4

23. $4 + 8 + 12 + 16$ $d=4$

$$\sum_{x=1}^{4} 4x$$

24. $3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{16} + \frac{3}{32}$ $r = \frac{1}{2}$

$$\sum_{x=1}^{5} 3\left(\tfrac{1}{2}\right)^{x-1}$$

$+1.5$

25. 6.5 $5 + 3\frac{1}{2} + 2 + \frac{1}{2} - 1$ $d = -1.5$

a_0

$$\sum_{p=1}^{5} -1.5p + 6.5$$

26. $-4 + 8 - 16 + 32 - 64$ $r = -2$

$$\sum_{m=1}^{5} -4(-2)^{m-1}$$

-3

27. -8 $-5 - 2 + 1 + 4 + 7 + 10$ $d=3$

a_0

$$\sum_{p=1}^{6} 3p - 8$$

28. $-1 + 3 - 9 + 27$ $r = -3$

$$\sum_{g=1}^{4} -(-3)^{g-1}$$

29-32: State whether each infinite geometric series is convergent or divergent. Then find the sum, if it exists.

29. $a_1 = 1, r = \frac{1}{2}$ convergent $|\frac{1}{2}| > 1$

$$S_\infty = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

30. $a_1 = 8, r = 2$ Divergent $|2| > 1$

No sum exists

31. $540 - 180 + 60 - 20 + \ldots$ $r = -\frac{1}{3}$ $|-\frac{1}{3}| > 1$

Convergent

$$S_\infty = \frac{540}{1-(-\frac{1}{3})} = \frac{540}{\frac{4}{3}} = 540 \cdot \frac{3}{4} = 405$$

32. $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ $r = \frac{1}{3}$ $|\frac{1}{3}| > 1$

convergent

$$S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2}$$

33-42: Find the sum of each series. *Circle your final answer.* G: Geometric A: Arithmetic

G 33. $\sum_{k=3}^{10} 4\left(\frac{3}{2}\right)^k$ $S_8 = \frac{13.5\left(1-\left(\frac{3}{2}\right)^8\right)}{\left(1-\frac{3}{2}\right)}$ $\frac{170235}{256}$

$a_1 = 4\left(\frac{3}{2}\right)^3 = 13.5$

A 34. $\sum_{r=2}^{50} 3 + 5r$ $S_{49} = \frac{49}{2}(13 + 253)$ $(r=2)$ $r=50$

6,517

G 35. $\sum_{p=0}^{12} (-2)^{p-1}$ $S_{13} = \frac{-\frac{1}{2}\left(1-(-2)^{13}\right)}{(1-(-2))}$

$a_1 = (-2)^{0-1} = (2)^{-1} = -\frac{1}{2}$ -1365.5

A 36. $\sum_{k=1}^{20} -2k - 3$ $S_{20} = \frac{20}{2}\left(\underset{k=1}{-5} + \underset{k=20}{-43}\right)$

-480

IYF! **37.** $a_1 = 6, a_6 = 21$ (Arithmetic)

$S_6 = \frac{6}{2}(6+21) = 81$

A **38.** $5 + 8 + 11 + 14 + \ldots$; find S_{22}

$d=3$

$a_{22} = 5+21(3) = 68$

$S_{22} = \frac{22}{2}(5+68) = 803$

G **39.** $2 - 1 + \frac{1}{2} - \frac{1}{4} \ldots$; find S_{10}

$r = -\frac{1}{2}$

$S_{10} = \frac{2(1-(-\frac{1}{2})^{10})}{(1-(-\frac{1}{2}))} = \frac{341}{256}$

A **40.** $a_1 = 2, d = 5, n=20$

$a_{20} = 2+19(5) = 97$

$S_{20} = \frac{20}{2}(2+97)$

990

G **41.** $a_1 = -2, r = \frac{-1}{2}$; find S_5

IYF!

$S_5 = \frac{-2(1-(-\frac{1}{2})^5)}{(1-(-\frac{1}{2}))} = \frac{-11}{8}$

G **42.** $a_7 = 128, r = 2$; find S_7

$a_1 \cdot r^6 = a_7$

$a_1(2)^6 = 128$

$64a_1 = 128$

$a_1 = 2$

$S_7 = \frac{2(1-(2)^7)}{(1-2)}$

254

Learning Target D	I can apply the properties of arithmetic and geometric sequences and series to solve real-life problems.

43-46: Use the appropriate formula(s) to solve each problem. Show detailed work to justify your answers.

43. Matthew is training to run a marathon. He runs 20 miles the first week of training and each week he increases the number of miles he runs by 4 miles. How many total miles did he run in 8 weeks of training?

$d=4$

$a_1 = 20$

$a_8 = 20+7(4) = 48$

$S_8 = \frac{8}{2}(20+48) =$ 272 miles

44. On its first swing, a pendulum travels 8 feet. On each successive swing, the pendulum travels $\frac{4}{5}$ the distance of its previous swing. What is the total distance traveled by the pendulum when it stops swinging?

r

$S_\infty = \frac{8}{1-\frac{4}{5}} = \frac{8}{\frac{1}{5}} =$ 40 feet

45. Colin purchased an antique dresser for $500. He kept the dresser for 20 years and has decided to sell it. If the average yearly rate of appreciation is 15%, what is the dresser currently worth?

a_0

incr. by .15

growth rate $1.15 = r$

$500, 575$

a_0

$a_{20} = 500(1.15)^{20}$

a_0

$8183.27

46. In the Righteous High School marching band, the students march in rows. There is one performer in the first row, three in the next row, and five in the third row. If there are 10 rows of performers in the halftime show, how many band students are marching?

Arithmetic

$1, 3, 5$ $d=2$

$a_1\ a_2\ a_3$

a_{10}

$S_{10} = \frac{10}{2}(1+19) =$ 100 students

$a_{10} = 1+(9)(2)$